# Communication Through Coherent Control of Quantum Channels

**Alastair A. Abbott**

based on joint work with
Julian Wechs, Dominic Horsman, Mehdi Mhalla and Cyril Branciard

QISS, Hong Kong, 14 January 2020

[arXiv:1810.09826]

UNIVERSITÉ DE GENÈVE

# Outline

Communication advantages from the Quantum Switch
Causal activation in the "depolarising switch"

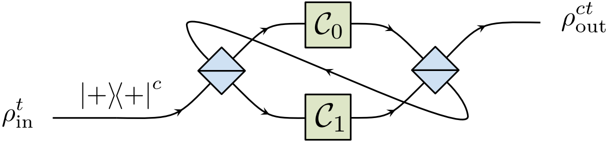
Communicating through coherently controlled channels
Defining coherently controlled channels
Communicating through controlled depolarising channels

Comparing control of channels and of their order

# The Quantum Switch

The quantum switch coherently controls the order in which two quantum channels are applied

- A new way to compose channels in a superposition of different orders




- How can the quantum switch be used as a resource for quantum communication?
  - Communication complexity: $\mathcal{C}_0(x)$ and $\mathcal{C}_1(y)$ interpreted as parties trying to compute some $f(x, y)$ with minimal communication [Guérin, Feix, Araújo, Brukner, PRL (2016)]
  - Communication capacity: increase capacity communicating *through* $\mathcal{C}_0$ and $\mathcal{C}_1$.

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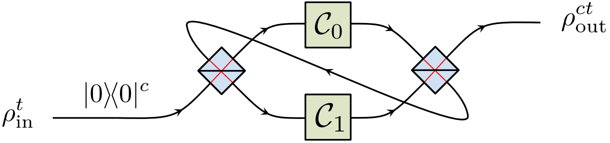


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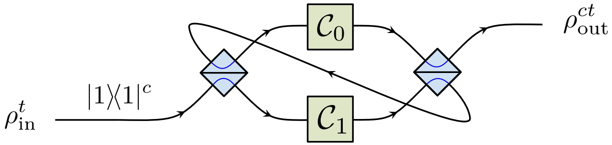


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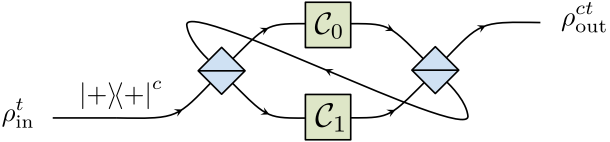


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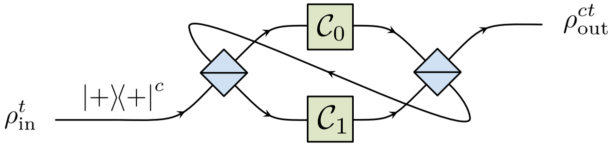


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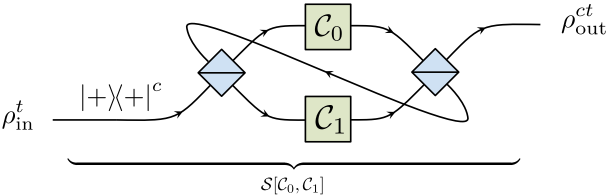


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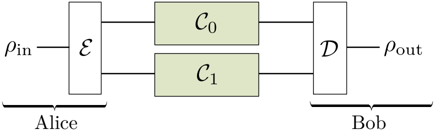
Imagine Alice and Bob wish to communicate and have access to some noisy channels

- Parallel composition

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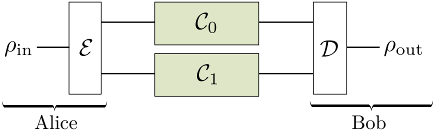


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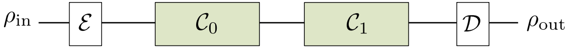


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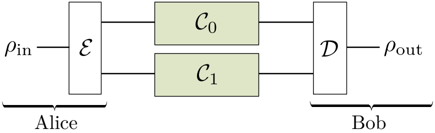


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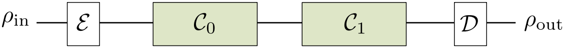


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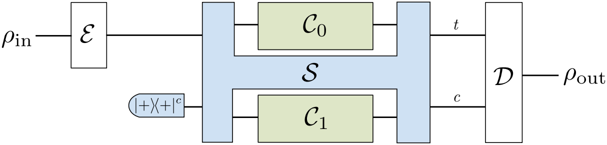

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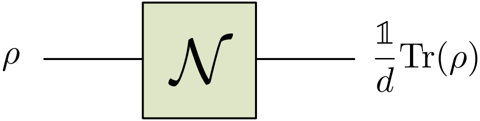
Ebler, Salek and Chiribella [PRL 120 (2018)] showed the quantum switch enables "causal activation" of channel capacity in this scenario

- Extreme case: classical information can be transmitted through two completely depolarising channels




- Depolarising channels can transmit no information, even when composed in a standard, causal manner
- When placed in a quantum switch, $\mathcal{S}[\mathcal{N}_0, \mathcal{N}_1]$ has nonzero capacity!
  - Experimentally realised: [Goswami, Romero, and White, arXiv:1807.07383; Guo et al., arXiv:1811.07526]

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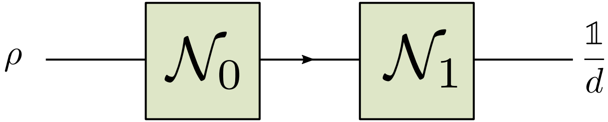


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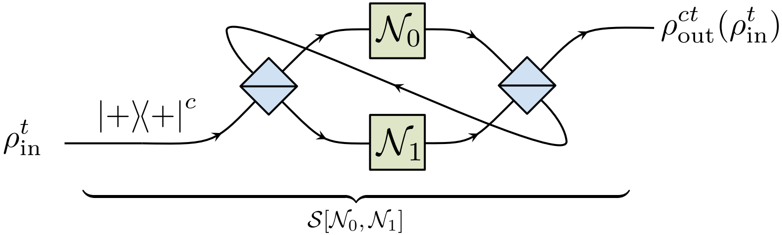


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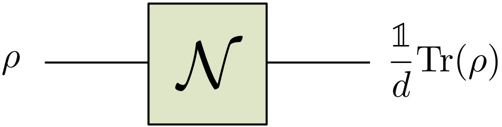


- For random choice $(U_i, U_j)$ system evolves under unitary

$$W_{ij} = U_j U_i \otimes |0\rangle\langle 0|^c + U_i U_j \otimes |1\rangle\langle 1|^c$$

- Output of global channel is

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$$= \frac{\mathbb{1}^c}{2} \otimes \frac{\mathbb{1}^t}{d} + \frac{1}{2}\left[\,|0\rangle\langle 1|^c + |1\rangle\langle 0|^c\,\right] \otimes \frac{1}{d^2}\rho^t_{\text{in}}$$

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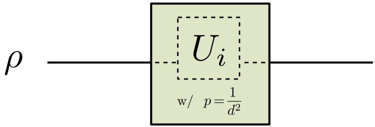


$$\frac{1}{d^2}\sum_i U_i \rho U_i^\dagger = \frac{\mathbb{1}}{d}$$

$\{\frac{1}{d}U_i\}_i$ Kraus operators for $\mathcal{N}$ (with $\{U_i\}_i$ orthogonal unitaries)

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  - Bob needs control to access it
- For qubits, Holevo information is $\chi(\rho_{\text{out}}^{ct}) = -\frac{3}{8} - \frac{5}{8}\log_2\frac{5}{8} \approx 0.05$

How does information get through?

- The $W_{ij}$ act jointly on the control and target, even if the controlled $U_{i/j}$ act only on the target

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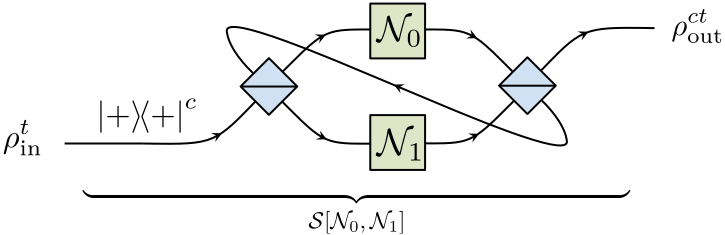
Consider the joint control-target state after having traversed half a quantum switch

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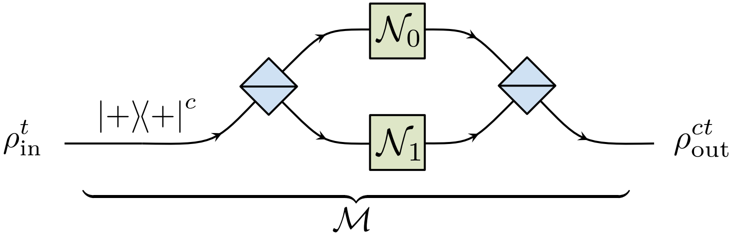


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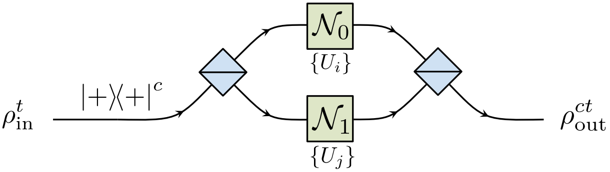


Take $\mathcal{N}_0, \mathcal{N}_1$ again as randomisation of $U_i, U_j$

- For random choice of unitaries $(U_i, U_j)$, system evolves under unitary

$$W'_{ij} = U_i \otimes |0\rangle\langle 0|^c + U_j \otimes |1\rangle\langle 1|^c$$

- Averaging over $(U_i, U_j)$ gives output

$$\rho_{\text{out}}^{ct} = \frac{\mathbb{1}^c}{2} \otimes \frac{\mathbb{1}^t}{d} + \frac{1}{2}\left[|0\rangle\langle 1|^c + |1\rangle\langle 0|^c\right] \otimes T\rho_{\text{in}}^t T^\dagger$$

with $T \coloneqq \frac{1}{d^2}\sum_i U_i$

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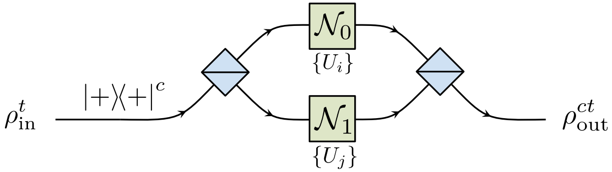


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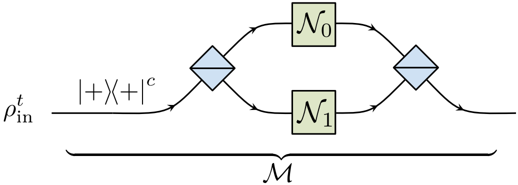


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- For orthogonal $U_i$, $T\rho^t_{\text{in}}T^\dagger \neq 0$ and depends on $\rho^t_{\text{in}}$, so some information is again transmitted!
- Unlike the quantum switch, setup has a clear causal and temporal order
- But $T = \frac{1}{d^2}\sum_i U_i$ depends on the orthonormal set $\{U_i\}_i$ chosen!
  - To speak about "coherently controlled channels", need further information about their implementation.

cf. Araújo, Feix, Costa, Brukner, NJP (2014); Thompson, Modi, Vedral, Gu, NJP (2018); Chiribella, Kristjánnson, PRSA (2019).

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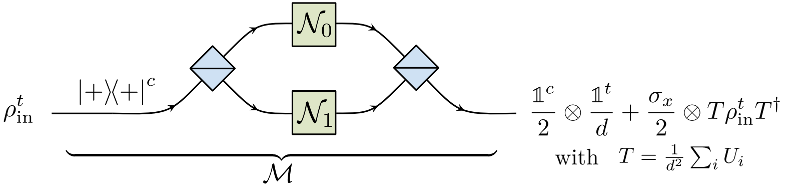


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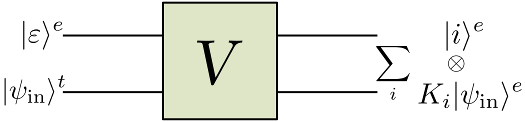
# General Purified Implementations

More generally, can always view a channel $\mathcal{C}$ as unitary interaction with some local environment $|\varepsilon\rangle^e$

- Given Kraus operators $\{K_i\}_i$ for $\mathcal{C}$, Stinespring purification:

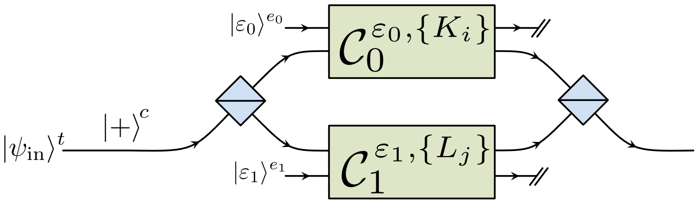



  giving joint target-environment evolution:

$$|\psi_{\mathsf{in}}\rangle^t \otimes |\varepsilon\rangle^e \to \sum_i K_i \, |\psi_{\mathsf{in}}\rangle^t \otimes |i\rangle^e := |\Phi_{\mathsf{out}}\rangle^{te}$$

- Tracing out environment gives $\mathrm{Tr}_e \, |\Phi_{\mathsf{out}}\rangle\langle\Phi_{\mathsf{out}}|^{te} = \mathcal{C}(|\psi_{\mathsf{in}}\rangle\langle\psi_{\mathsf{in}}|^t)$

# Calculating the Channel Dependence




Coherently controlling the unitary purified channels and tracing out the environments gives joint control-target output

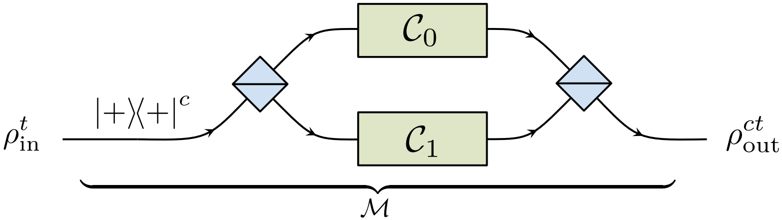
$$\rho_{\mathsf{out}}^{ct} = \frac{1}{2}\big[\,|0\rangle\!\langle 0|^c \otimes \mathcal{C}_0(\rho_{\mathsf{in}}^t) + |1\rangle\!\langle 1|^c \otimes \mathcal{C}_1(\rho_{\mathsf{in}}^t)\big] + \frac{1}{2}\big[|0\rangle\langle 1|^c \otimes T_0 \rho_{\mathsf{in}}^t T_1^\dagger + |1\rangle\langle 0|^c \otimes T_1 \rho_{\mathsf{in}}^t T_0^\dagger\big]$$

with $T_0 \coloneqq \sum_i \langle \varepsilon_0 | i \rangle K_i$ and $T_1 \coloneqq \sum_j \langle \varepsilon_1 | j \rangle L_j$.

# Coherent Control of Quantum Channels

Output depends on the transformation matrices $T_0$ and $T_1$

- Induced global channel is thus $\mathcal{M}[\mathcal{C}_0, T_0, \mathcal{C}_1, T_1]$

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  - Think about "global" phases becoming "relative" in interferometers
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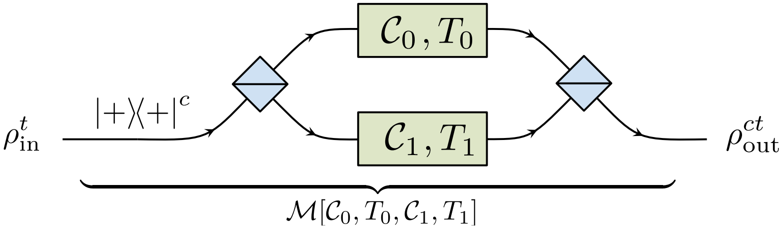


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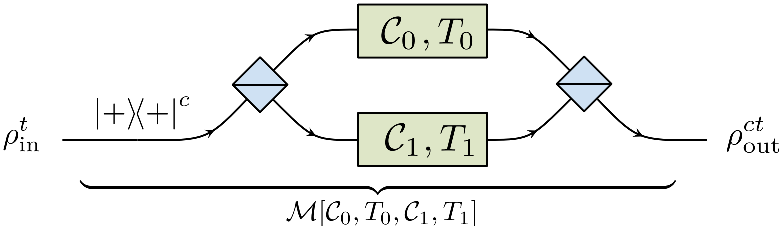


CPTP map $\mathcal{C}$ must be supplemented by $T$ to meaningfully describe the coherent control of the channel – i.e., the "channel implementation" $(\mathcal{C}, T)$

- Equivalent to "vacuum-extended channels" of Kristjánsson and Chiribella [PRSA, 2019]

# Characterising Possible Transformations

For a given CPTP map $\mathcal{C}$, what transformation matrices $T$ can one have?

For a unitary $\mathcal{U} : \rho \mapsto U\rho U^\dagger$:

- One can have $T = \alpha U$ with $\alpha \in \mathcal{C}$, $|\alpha| \leq 1$
- "Implementation details" are just the phase w.r.t. some reference

For arbitrary $\mathcal{C}$, can characterise $T$ in terms of the Choi state $C$ of $\mathcal{C}$ and its (pseudo)inverse

- For a (completely depolarising channel), one can have any $T$ satisfying $\mathrm{Tr}[T^\dagger T] \leq \frac{1}{d}$
- Allows optimisation over possible "implementations" of a channel

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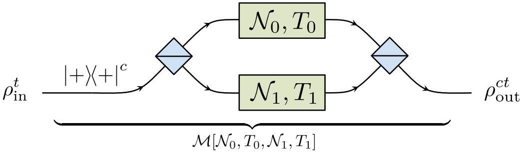
- Allows optimisation over possible "implementations" of a channel

# Coherent Control of Depolarising Channels




How much information can one communicate through coherently controlled depolarising channels?

- Three cases of interest saturating $\mathrm{Tr}[T^\dagger T] \leq \frac{1}{d}$:
- Taking $K_i = \frac{1}{d} U_i$ and $|\varepsilon\rangle = \sum_{i=0}^{d^2-1} \frac{1}{d} |i\rangle$ gives $T = \frac{1}{d^2} \sum_i U_i$
  - Recover result of randomisation over $(U_i, U_j)$
  - Gives Holevo information $\chi(\mathcal{M}[\mathcal{N}_0, T_0, \mathcal{N}_1, T_1]) \approx 0.12$
- Taking $|\varepsilon\rangle = |0\rangle$ and $U_0 = \frac{1}{d}\mathbb{1}$ for each channel gives $T = \frac{1}{d}\mathbb{1}$
  - One recovers precisely the output of the depolarising quantum switch
  - Recall for that case, $\chi(\mathcal{M}[\mathcal{N}_0, T_0, \mathcal{N}_1, T_1]) \approx 0.05$
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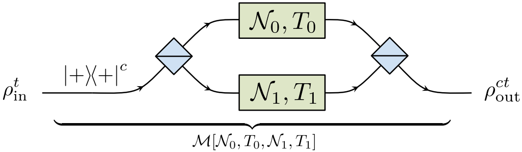


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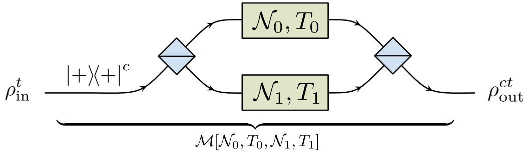


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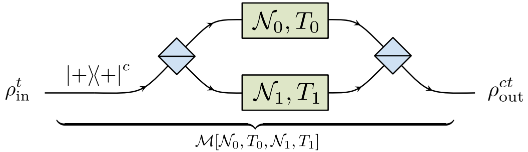


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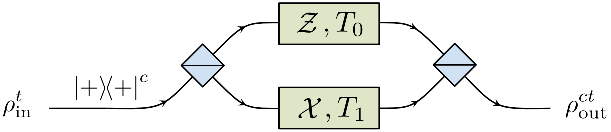
# Activation of Quantum Capacity

- What about quantum capacity?
  - Salek, Ebler and Chiribella [1809.06655] showed the quantum switch can activate quantum capacity for communication through dephasing channels




- Again, coherent control of channels activates quantum capacity as well
  - The quantum switch only shown to activate capacity for partially dephasing channels
  - Coherent control activates capacity even for maximally completely channels

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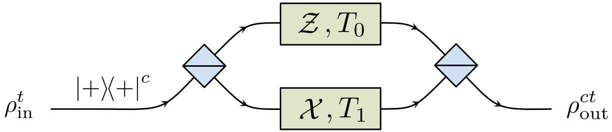


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# Communication Advantages and Causality

Does this mean these effects can't be considered "causal activation"?

- Coherent control activates classical capacity without any causal indefiniteness
- Can this be compared directly to the quantum switch?
  - Statement about $(\mathcal{C}, T)$ pairs: activation depends on channel implementation
- In operational "black-box" framework, reasonable to treat these as primitive objects, even if in many situations the CPTP map $\mathcal{C}$ is sufficient

- Chiribella et al. [1810.10457] showed there exist some scenarios where $\mathcal{S}[\mathcal{C}_0, \mathcal{C}_1]$ has maximal quantum capacity while $\mathcal{C}_0$ and $\mathcal{C}_1$ have zero capacity and that this is impossible with coherent control of channels alone

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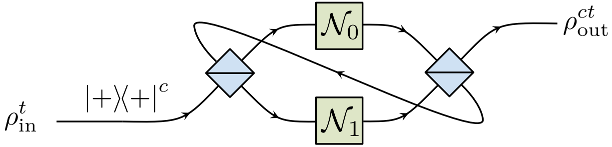
What is the right framework for formulating communication advantage here?

- Is the control acting as a side-channel?
- Can't compare with arbitrary processes
- Guérin, Rubino and Brukner [PRA 2019]: "direct pure processes"

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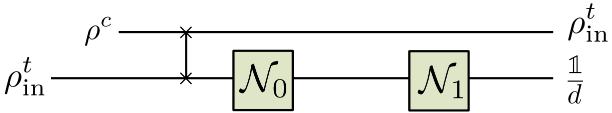


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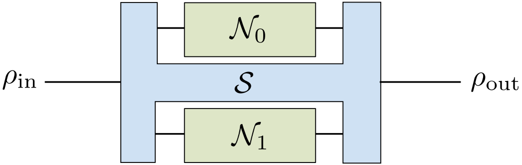


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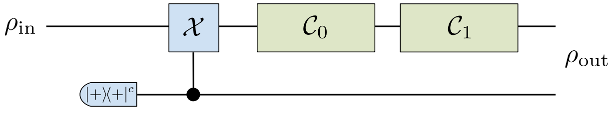


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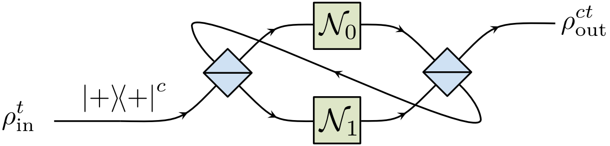


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# Relevant and Reasonable Resources

Resource theory approach of Kristjánsson, Salek, Ebler and Chiribella:

**Side-channel generating operation [arXiv:1910.08197]**

An operation $\mathcal{S} : (\mathcal{C}_0, \mathcal{C}_1) \mapsto \mathcal{S}[(\mathcal{C}_0, \mathcal{C}_1)]$ generates a side channel if there exist $\mathcal{E}, \mathcal{D}$ such that for all $(\mathcal{C}_0, \mathcal{C}_1)$

$$\mathcal{D} \circ \mathcal{S}[(\mathcal{C}_0, \mathcal{C}_1)] \circ \mathcal{E} = \mathcal{M},$$

where $\mathcal{M}$ is a fixed channel with nonzero capacity.

Requirement: A resource theory for communication should forbid side-channel generating operations

- Rules out direct pure processes
- Excludes sender from accessing control system of quantum switch
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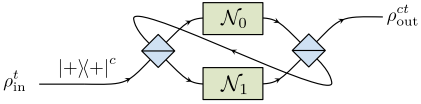
# Relevant and Reasonable Resources

Apparent tension between:

- Desire for interesting, non-trivial resource theory
- Physical resources required to implement the different composition operations

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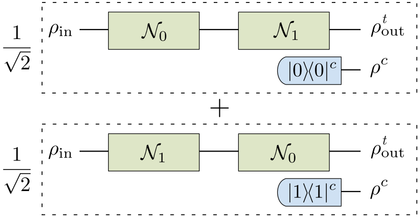
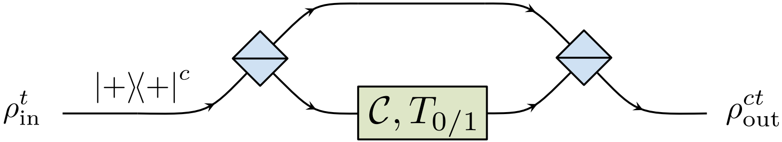



- Relevance of resource theory seems to depend on the spatio-temporal implementation of composition operations

# Exploiting Coherent Control of Channels

If one can control coherently the use of channels in a black-box manner, why not exploit it?

- Implementation dependence a subtlety, but opens up new possibilities
  - E.g., discrimination of different implementations of a channel




- Two implementations of $\mathcal{C}$ with transformation matrices $T_0$ and $T_1$ induce two different global channels $\mathcal{M}_{T_1}$ and $\mathcal{M}_{T_2}$
- If chosen with equal priors, can discriminate with probability

$$\frac{1}{2}\left(1 + \tfrac{1}{2}\|T_0 - T_1\|_2\right)$$

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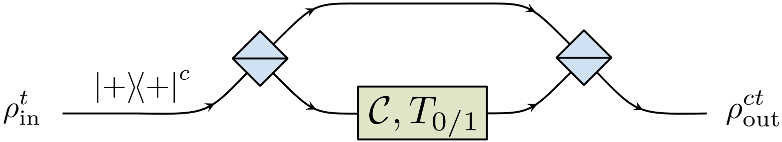


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# Exploiting Coherent Control of Channels

What can we gain from treating coherently controllable channels as an operational primitive?

- Much more work to be done to understand what advantages such an approach could entail
- Adds to more general call to extend the standard circuit approach to experimentally conceivable situations
  - E.g., Araújo at al., NJP 16 (2014); Portmann et al., IEEE Trans. IT 63 (2017); Thompson, Modi, Vedral, Gu, NJP (2018)
  - Chiribella and Kristjánsson's approach a first step [Proc. R. Soc. A 475 (2019)]

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# The End

Thank you!

[arXiv:1810.09826]

Further reading:

- Causal activation paper: Ebler, Salek, and Chiribella, PRL **120**, 120502 (2018)
- With quantum information: Salek, Ebler, and Chiribella, arXiv:1809.06655
- Activation impossible with control of path only: Chribella et al., arXiv:1810.10457
- Other ways to cheat activation: Guérin, Rubino, and Brukner, PRA **99**, 062317 (2019)
- More general model: Chiribella and Kristjánsson, Proc. R. Soc. A **475** (2019)
- Resource theory approach: Kristjánsson, Salek, Ebler and Chiribella, arXiv:1910.08197