Communication Through Coherent Control of Quantum Channels

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joint work with
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Université de Genève, Geneva, Switzerland

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UNIVERSITÉ DE GENÈVE

Outline

Communication advantages from the Quantum Switch

Causal activation in the “depolarising switch”

Communicating through coherently controlled channels

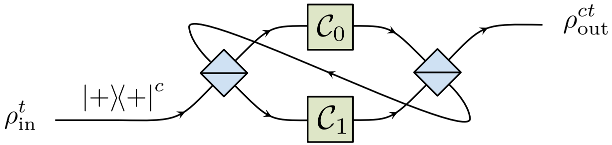
Coherently controlled depolarising channels

The role of causality in communication advantages

The Quantum Switch

The quantum switch coherently controls the order in which two channels are applied

- A new way to compose channels in a superposition of different orders



- The quantum switch provides novel advantages in quantum information
  - E.g., in determining properties of $\mathcal{C}_0$ and $\mathcal{C}_1$ [Chiribella, PRA 86 (2012)]
- Can it also be seen as a resource for quantum communication?
  - The QS induces a new global channel $\mathcal{S}[\mathcal{C}_0, \mathcal{C}_1]$ from Alice to Bob

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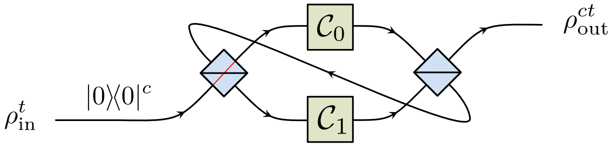

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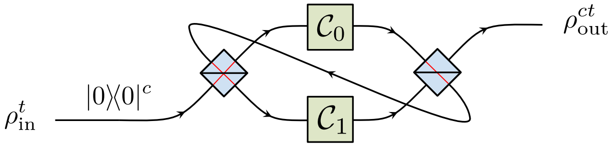

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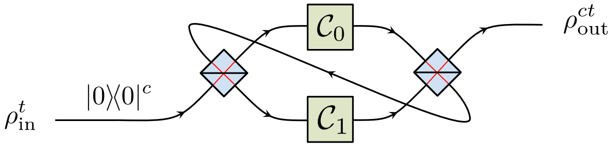

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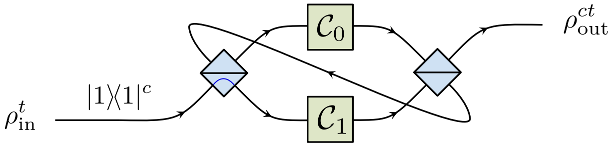

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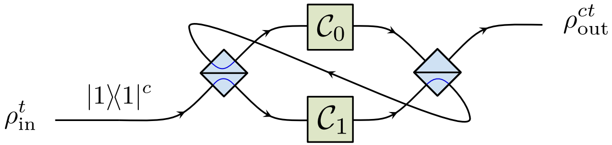

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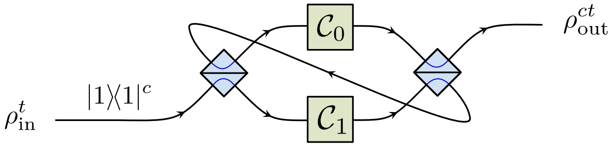

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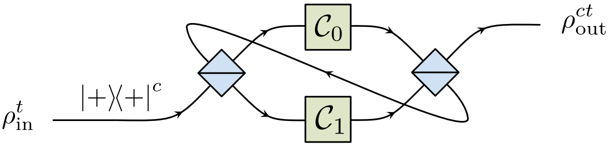

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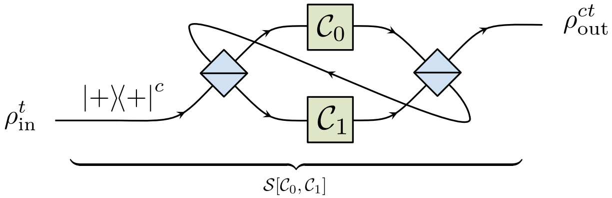

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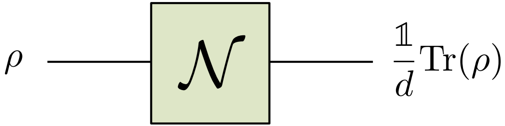
Depolarising Quantum Switch

Ebler, Salek and Chiribella [PRL 120 (2018)] showed the quantum switch enables a type of "causal activation" of channel capacity

- It allows (classical) information to be transmitted through two completely depolarising channels



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  - Experimentally realised already [Goswami, Romero, and White, arXiv:1807.07383; Guo et al., arXiv:1811.07526]

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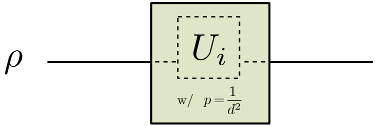

$$\frac{1}{d^2}\sum_i U_i \rho U_i^\dagger = \frac{\mathbb{1}}{d}$$

$\{\frac{1}{d}U_i\}_i$ Kraus operators for $\mathcal{N}$ (with $\{U_i\}_i$ orthogonal unitaries)

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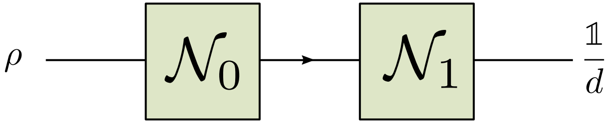

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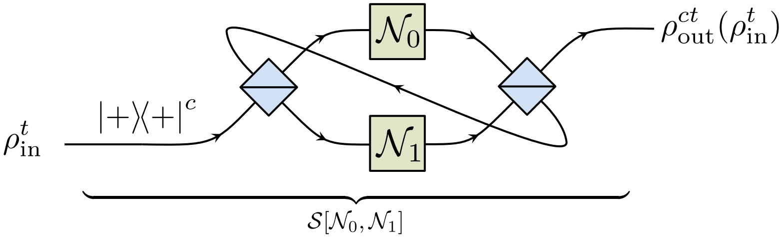

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How Does it Work?

For random choice (U_i, U_j) system evolves under unitary

$$W_{ij} = U_j U_i \otimes |0\rangle\langle 0|^c + U_i U_j \otimes |1\rangle\langle 1|^c$$

- Output of global channel is

$$\mathcal{S}[\mathcal{N}_0, \mathcal{N}_1](\rho_{\text{in}}^t) = \frac{1}{d^4} \sum_{ij} W_{ij} (\rho_{\text{in}}^t \otimes |+\rangle\langle +|^c) W_{ij}^\dagger$$
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- For qubits, Holevo information is $\chi(\rho_{\text{out}}^{ct}) = -\frac{3}{8} - \frac{5}{8} \log_2 \frac{5}{8} \approx 0.05$

Should we attribute this to the indefinite causal order of the quantum switch?

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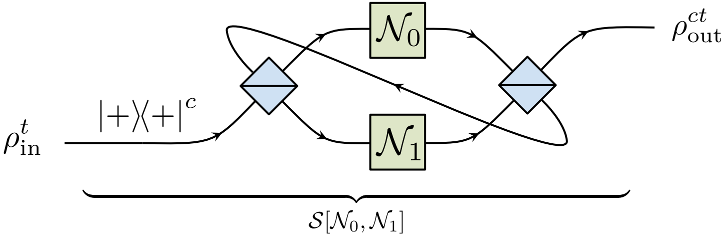
Cutting The Switch in Half

Consider the joint control-target state after having traversed half a quantum switch



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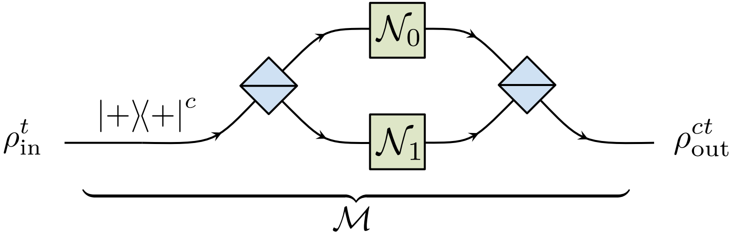

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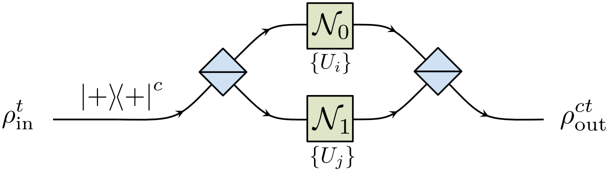
Analysing the "Half-Switch"



To compute ρ_{out}^{ct} consider the same implementation of $\mathcal{N}_0, \mathcal{N}_1$

- For random choice of unitaries (U_i, U_j), an input $|\psi_{\text{in}}\rangle^t$ gives the output

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with $T := \frac{1}{d^2}\sum_i U_i$ and $\rho_{\text{in}}^t := |\psi_{\text{in}}\rangle\langle\psi_{\text{in}}|^t$

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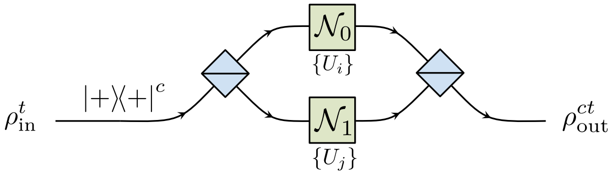

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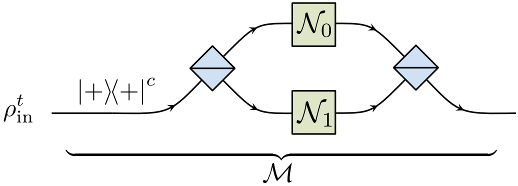

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with $\quad T = \frac{1}{d^2}\sum_i U_i$

- $T\rho_{\text{in}}^t T^\dagger \neq 0$ and depends on ρ_{in}^t, so some information is again transmitted!
  - Unlike the quantum switch, setup has a clear causal and temporal order
- Requires coherent control: if the control is decohered, second term is lost
- Holevo capacity actually *larger* than for the depolarising quantum switch
  - For qubits, taking the U_i as the Pauli matrices one calculates $\chi(\mathcal{C}) \approx 0.12$ (compared to ≈ 0.05 for the switch)

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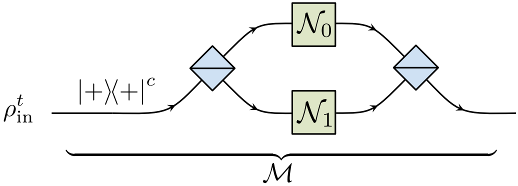

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Dependence on Channel Implementation

Note that $T = \frac{1}{d^2}\sum_i U_i$ depends on the orthonormal set $\{U_i\}_i$ chosen!

- More generally, Stinespring purification for any Kraus operators $\{K_i\}_i$

- Unitary interaction between target $|\psi_{\text{in}}\rangle^t$ and local environment $|\varepsilon\rangle^e$:

$$|\psi_{\text{in}}\rangle^t \otimes |\varepsilon\rangle^e \to \sum_i K_i |\psi_{\text{in}}\rangle^t \otimes |i\rangle^e := |\Phi_{\text{out}}\rangle^{te}$$

- Tracing out environment gives $\text{Tr}_e |\Phi_{\text{out}}\rangle\langle\Phi_{\text{out}}|^{te} = \mathcal{C}(|\psi_{\text{in}}\rangle\langle\psi_{\text{in}}|^t)$

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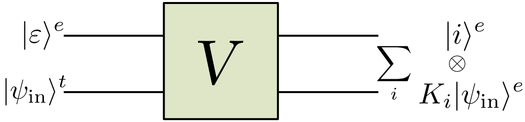

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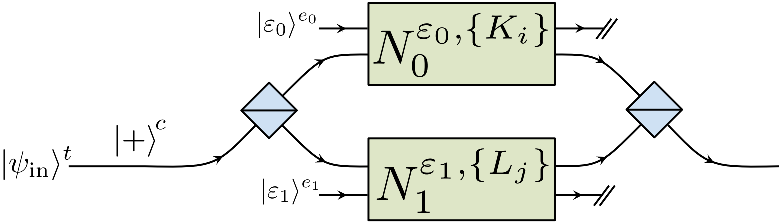
Calculating the Channel Dependence



Applying the unitary purified channels and tracing out the environments gives joint control-target output

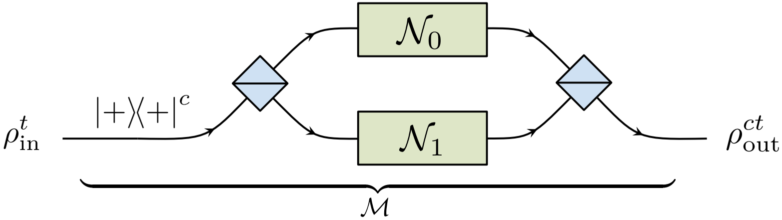
$$\rho_{\mathsf{out}}^{ct} = \frac{\mathbb{1}^c}{2} \otimes \frac{\mathbb{1}^t}{d} + \frac{1}{2}\left[|0\rangle\langle 1|^c \otimes T_0 \rho_{\mathsf{in}}^t T_1^\dagger + |1\rangle\langle 0|^c \otimes T_1 \rho_{\mathsf{in}}^t T_0^\dagger\right]$$

with $T_0 \coloneqq \sum_i \langle \varepsilon_0 | i \rangle K_i$ and $T_1 \coloneqq \sum_j \langle \varepsilon_1 | j \rangle L_j$

Coherent Control of Quantum Channels

Output depends on the transformation matrices T_0 and T_1

- Induced global channel is thus $\mathcal{M}[\mathcal{N}_0, T_0, \mathcal{N}_1, T_1]$



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  - Think about "global" phases becoming "relative" in interferometers
- Here we have a deeper dependence on the full purification
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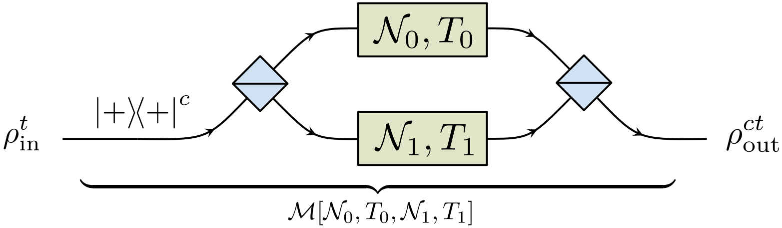

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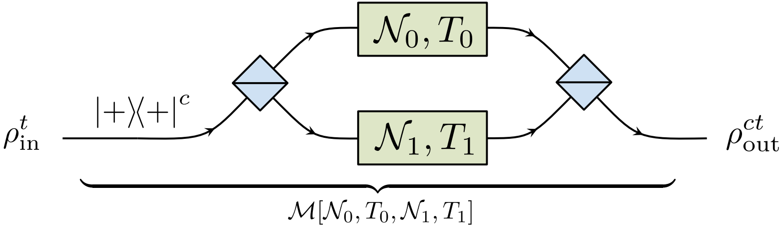
Characterising Possible Outputs



How much information can be transmitted?

- For a depolarising channel, we prove that one must have $\mathrm{Tr}[TT^\dagger] \leq \frac{1}{d}$
  - Can characterise obtainable T for any CPTP map $\mathcal{C}$
- Optimal is $\chi(\mathcal{M}[\mathcal{N}_0, T_0, \mathcal{N}_1, T_1]) = \frac{1}{d}\log_2\frac{5}{4}$, which is ≈ 0.16 for qubits
  - Notice that this decreases with d!

Communication Advantages and Causality

So what about the quantum switch and causal activation?

- Can activate classical capacity with coherent control and no causal indefiniteness
  - Similar approach of error filtration previously described by Gisin et al. [PRA 72 (2005)]
- Effect not restricted to classical capacity: a similar effect for quantum capacity was shown [Salek et al. arXiv:1809.06655] which can also be obtained with simple coherent control

Chiribella et al. [arXiv:1810.10457] showed there exist some scenarios where $\mathcal{S}[\mathcal{C}_0, \mathcal{C}_1]$ has maximal quantum capacity while $\mathcal{C}_0$ and $\mathcal{C}_1$ have zero capacity and that this is impossible with coherent control of paths alone

How much is due to coherent control vs. causal indefiniteness?

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- Effect not restricted to classical capacity: a similar effect for quantum capacity was shown [Salek et al. arXiv:1809.06655] which can also be obtained with simple coherent control

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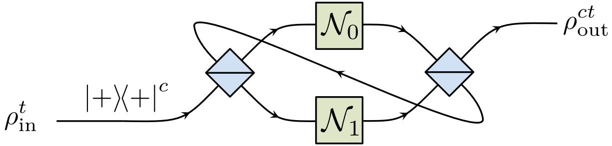
How much is due to coherent control vs. causal indefiniteness?

What is the Right Framework?

Several aspects of the question still need further clarification

- The control system is unaffected by noisy channels and is "transmitted" perfectly
  - Is it acting as a side-channel?



- Chiribella and Kristjánsson [PRSA 475 (2019)] proposed a more general framework to treat both coherent control of channels and their order
  - Proposed explicit encoding procedure with conditions to avoid such a possibility
- Appropriate in certain scenarios but distinction between encoding/transmission stages not always clear [cf. Guérin et al., PRA 99 (2019)].

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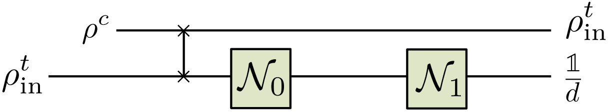

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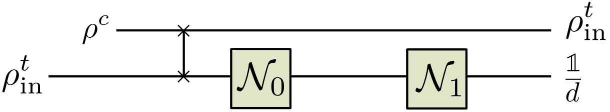

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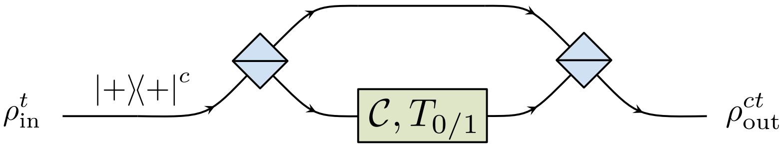
Exploiting Coherent Control of Channels

If one can control coherently the use of channels in a black-box manner, why not exploit it?

- Implementation dependence a subtlety, but opens up new possibilities
  - E.g., discrimination of different implementations of a channel



- Possible applications in error correction or security?
- More general call to extend the standard circuit approach to experimentally conceivable situations
  - E.g., Araújo at al., NJP 16 (2014); Portmann et al., IEEE Trans. IT 63 (2017); Chiribella and Kristjánsson, Proc. R. Soc. A 475 (2019)

The End

Thank you!

[arXiv:1810.09826]

Further reading:

- Causal activation paper: Ebler, Salek, and Chiribella, PRL **120**, 120502 (2018)
- With quantum information: Salek, Ebler, and Chiribella, arXiv:1809.06655
- Activation impossible with control of path only: Chribella et al., arXiv:1810.10457
- Other ways to cheat activation: Guérin, Rubino, and Brukner, PRA **99**, 062317 (2019)
- More general model: Chiribella and Kristjánsson: Proc. R. Soc. A **475** (2019)