L'aléatoire et l'imprévisibilité au cœur de la mécanique quantique

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Randomness and Quantum Mechanics

Randomness is often seen as an integral part of quantum mechanics, and plays a key role in new "quantum technologies"

"*. . . quantum randomness in general has a very different and intrinsic or inherent nature. Namely, even if the state of the system is pure and we know it exactly, the predictions of quantum mechanics are intrinsically probabilistic and random! Accepting quantum mechanics means making an assumption that it is correct, and consequently intrinsically random.*"[1]

"*Any classical system admits in principle a deterministic description and thus appears random to us as a consequence of a lack of knowledge about its fundamental description. Quantum theory is, on the other hand, fundamentally random.*"[2]

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[1] M. Bera *et al.*, arXiv:1611.02176, 2016.

[2] S. Pironio *et al.*, Nature 464 (2010), p. 1021.

Outline

Quantum Primer

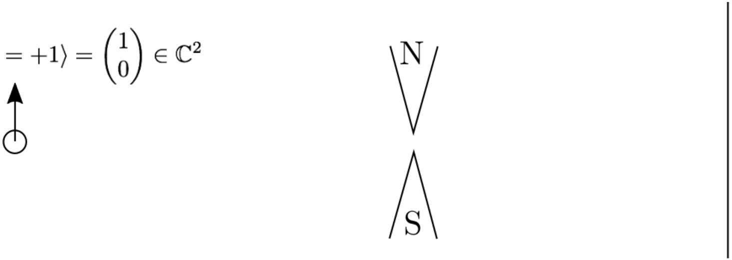
Quantum Indeterminism
- Interpreting the Born rule
- No-go theorems against constrained determinism

Quantum Randomness
- Indeterminism and forms of randomness
- Quantum randomness as unpredictability

Unpredictability and Randomness
- Formalising Unpredictability

Quantum Basics

$$|S_z = +1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \in \mathbb{C}^2$$



- States specified by sure propositions about measurable quantities
  - Formally, complex vectors in Hilbert space
- Measurement contexts are maximal sets of mutually exclusive propositions (projection operators)
- Incompatibility of measurements: $S_x \cdot S_z \neq S_z \cdot S_x$
- Quantum logic: non-distributive orthomodular lattice of propositions

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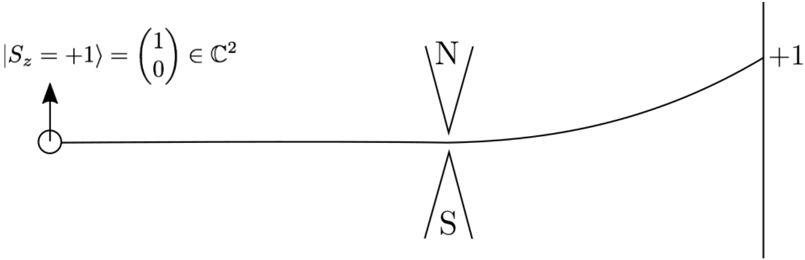

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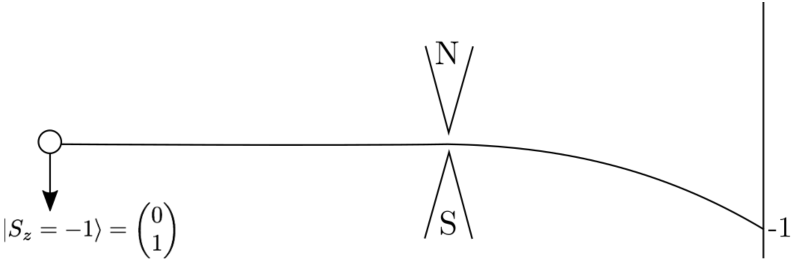

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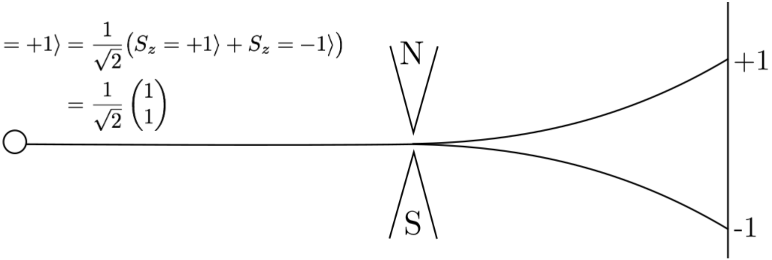
Quantum Basics

$$|S_x = +1\rangle = \frac{1}{\sqrt{2}}\big(S_z = +1\rangle + S_z = -1\rangle\big)$$
$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$



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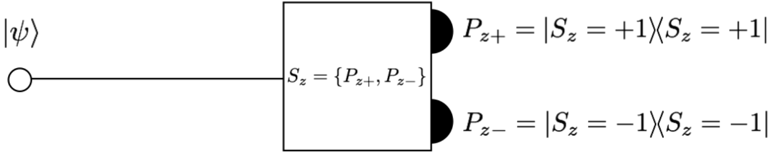

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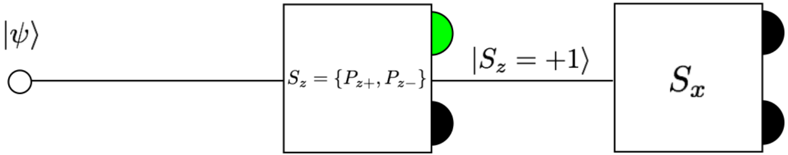

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The Born Rule

The Born rule links the abstract quantum state to the concrete observation of outcomes

The Born rule

The probability of observing outcome a_i upon measurement of the context $\{|a_i\rangle\langle a_i|\}_i$ on a state $|\psi\rangle$ is $p(a_i|\psi) = |\langle a_i|\psi\rangle|^2$. Following the observation of outcome a_i, the system 'collapses' to the state $|a_i\rangle$.

- This rule only tells us how to assign probabilities to observations, but is silent towards its physical *interpretation*
  - Large conceptual gap between this and the supposed 'intrinsic randomness' of quantum measurements
- Nor does it say anything about how this state collapse occurs
  - The standard dynamics of a quantum state are unitary and *reversible*

Interpreting the Born Rule

Epistemic interpretation

- The probabilities express only our ignorance as to the true physical state of the system, and statements about the quantum state should be interpreted as referring to physical ensembles
  - Einstein & Schrödinger notably favoured such an approach
  - "*The statistical interpretation due to Born . . . allows, however, no real description for the individual system, rather only statistical assertions concerning ensembles of systems.*" (Einstein, 1953)

Ontic interpretation

- The probabilities must be interpreted as physical propensities, and thus quantum measurement is fundamentally chancy and indeterministic
  - "*I myself tend to give up determinism in the atomic world.*" (Born, 1926)

Interpreting the Born Rule (2)

This ontic interpretation has become predominant, particularly in the modern information-theoretic approach to QM

Eigenstate–Eigenvalue Link[3]

A system in a state $|\psi\rangle$ has a definite value of an observable property A if and only if $|\psi\rangle$ is an eigenstate of A.

- This interpretation represents a radical departure from classical determinism; what basis do we have for believing this is necessary?
- And does this interpretation explain common accounts of quantum randomness?
  - Need to look at *what* randomness is and how it's related to indeterminism

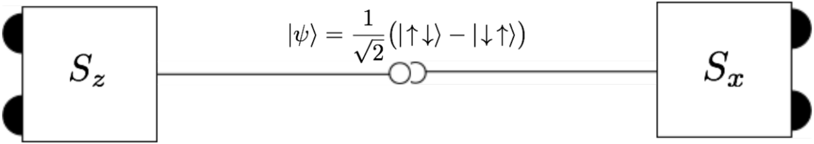
[3] M. Suárez, Br. J. Phil. Sci. 55 (2004), p. 222.

EPR's Incompleteness Argument

Einstein, Podolsky, and Rosen (1935) constructed an ingenious example to try and argue that quantum mechanics is incomplete, and that this indeterministic interpretation incorrect

- (Completeness) "*Every element of the physical reality must have a counter part in the physical theory.*"
- "*If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*"
- (Locality [implicit]) No physical influence can travel faster than the speed of light

EPR's Incompleteness Argument (2)



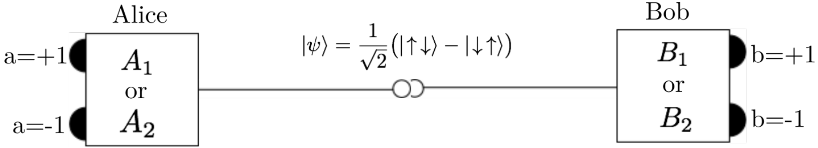
"*Either (1) the quantum mechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality.*"

EPR "*conclude that the quantum-mechanical description of physical reality given by wave functions is not complete.*"

- Can quantum mechanics be "completed"?

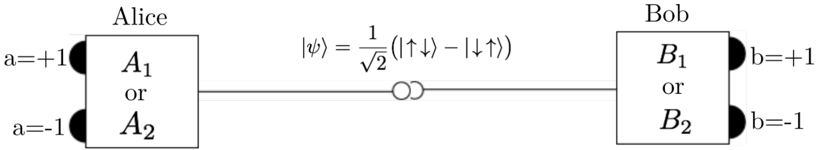
Bell's Theorem and Inequalities

This problem was taken up again in the 1960s, first by von Neumann and then Bell, Kochen and Specker



- Assume a hidden parameter λ completely determines the observable properties of the system
  - Preparation of $|\psi\rangle$ corresponds to preparing a $\lambda \in \Lambda_\psi$ that completely determines all measurement outcomes
  - The Born rule should be recovered as an ensemble average
- Locality: The measurement outcomes are a function only of λ and the *local*, freely made, measurement choice

Bell's Theorem and Inequalities (2)



Any "locally realist" theory must satisfy the Bell inequality

$$\langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle \leq 2$$

but quantum mechanics violates it

- Can be experimentally tested and the violation confirmed
- This rules out, at a statistical level, local realism for entangled states: determinism can only be saved if locality is abandoned
- Note that, if a violation is observed, this is an indicator of nonclassical physical behaviour, whether or not quantum mechanics is complete and correct

The Kochen-Specker Theorem

Kochen and Specker simultaneously showed, via a different form of argument, that the contradiction with determinism occurs even at the level of the Hilbert space representation, *prior* to measurement

- Assumes *non-contextuality* of hidden variables, a generalisation of locality
- Applies to *individual* quantum systems

Eigenstate-Eigenvalue Link

A system in a state $|\psi\rangle$ has a definite value of an observable property A if and only if $|\psi\rangle$ is an eigenstate of A.

The Bell and KS theorems justify the E-E link by proving quantum value indefiniteness under weak assumptions about how deterministic theories must be

Value Indefiniteness and Indeterminism

Important to note that these conclusions nonetheless not ‘unconditional’, as quantum randomness often claimed to be: they rest on the assumptions of the Bell-KS theorems

- Explicit deterministic theories exist, but are necessarily nonclassical, e.g. Bohmian mechanics
- Bell’s theorem shows this value indefiniteness is not a consequence of the formal structure of QM, and must hold for any extension thereof

Bell-KS value indefiniteness can easily be seen to be precisely a kind of indeterminism: the outcome of the measurement is not determined prior to the act of measurement

Physical Indeterminism

Indeterminism is defined as the logical negation of determinism

"*Une intelligence qui, à un instant donné, connaîtrait toutes les forces dont la nature est animée et la situation respective des êtres qui la composent, si d'ailleurs elle était suffisamment vaste pour soumettre ces données à l'analyse, embrasserait dans la même formule les mouvements des plus grands corps de l'univers et ceux du plus léger atome ; rien ne serait incertain pour elle, et l'avenir, comme le passé, serait présent à ses yeux.*" (Laplace, 1814)

Determinism (Earman, Montague)

A theory is deterministic if the state of a (closed) system at time t_0 uniquely determines its future state for all $t > t_0$.

Is Randomness Indeterminism?

Does quantum indeterminism amount to quantum randomness? More generally, can randomness be equated with indeterminism?

- Such a view was commonplace until recently dates perhaps to Laplace's assertion than "rien ne serait incertain" (and thus random) in a deterministic world
- This identification is still often implicitly adopted in the physical sciences

"*The result is completely random because in such a measurement the elementary system carries no information whatsoever about the measurement result.*"[4]

- It is useful to introduce chance as an intermediary concept since literature largely based about its relation to indeterminism and randomness

[4] A. Zeilinger, Found. Phys 29 (1999), p. 636.

Indeterminism, Chance and Randomness

It is instructive to analyse this identification in two parts:

(IC) Something happens by chance iff it is indeterministic

(CT) Something is random iff it happens by chance

What is meant by 'chance'?[5]

- Refers to possibility (as opposed to necessity), e.g. as "graded possibility" (Leibniz)
- Has a quantitative aspect, representable as probabilities
- Regulates rational belief in line with Lewis' Principal Principle:

$$\mathsf{Cr}(A|\text{“}P(A|E) = x\text{”} \wedge E) = x$$

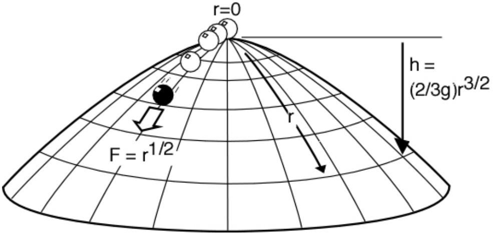
- Should relate to frequency of realisation in similar circumstances

[5] E.g. J. Schaffer, Brit. J. Phil. Sci 54 (2003), 27–41; D. Mellor, Aus. J. Phil. 78 (2000), 16–27.

IC: Indeterminism and Chance

IC has been challenged in several ways in recent years

- Arguments for deterministic chance, mostly based on existence of probabilities that objectively play the role of chance in theories despite underlying determinism[6]
- Indeterminism in (even classical) physics that doesn't play the role of chance, e.g. Norton's dome



[6] Eg. B. Loewer, Stud. Hist. Phil. Mod. Phys. 32 (2001), pp. 609–620; Eagle, Noûs 45:2 (2011), 269–299.

Quantum Measurement and Chance

The negative definition of indeterminism leaves the future "*completely unspecified. [...] Randomness such as described by the Born rule of quantum mechanics, on the other hand, is not a matter of 'anything goes', but a highly constrained affair: exactly one of a number of possible outcomes will occur, and the probabilities of all the outcomes are given beforehand.*"[7]

- So quantum measurements seems to fit the bill for being considered (objectively) chancy

Müller goes on to argue that quantum randomness is precisely this kind of constrained indeterminism

- This argument, and other more casual ones, for quantum randomness nonetheless appeal implicitly to the CT

[7] T. Müller, *(Quantum) randomness as limited indeterminism* (2015).

The Commonplace Thesis

(CT) Something is random iff it happens by chance

For CT to be tenable, chance needs to satisfy what we expect of randomness

The qualities of randomness can be divided into two groups: those dealing with objects, and those dealing with processes

- Product randomness: is a particular sequence of results of coin flips random?
- Process randomness: is the coin-flip process random?
- Chance is clearly a process notion, so already clear that the CT can only directly apply to a process notion of randomness

Product Randomness

The notion of randomness for finite and infinite strings has been rigorously formalised as a form of disorder or patternlessness

- Borel and von Mises made important historic contributions
- Kolmogorov (1965), Martin-Löf (1966), Chaitin (1966): All finite prefixes of a sequence should by incompressible by a universal prefix-free Turing machine
  - Some difficulty applying to (short) finite strings, and particularly *individual* outcomes
  - Impossible to verify whether a given sequence algorithmically random
  - Absolute algorithmic randomness provably impossible

Product randomness is nonetheless a productive and useful concept, and deserves to be considered a legitimate form of randomness

Process Randomness

Less consensus, but often seen as related to unpredictability

- Idea already present in Laplacian indeterminism, and taken up by others including Popper[8]
  - Need to be able to talk about individual random events
- Eagle goes a step further defined randomness as “maximal unpredictability”

Defining randomness as a form of unpredictability inserts a subjective element into randomness: unpredictable for whom? And under what conditions?

Many paradigmatic random events are (at least partially) deterministic:

- coin tossing, trajectories of chaotic systems

[8] Br. J. Phil. Sci, 1 (1950), 117.

The Commonplace Thesis?

Both product and process randomness seem legitimate and distinct

- Attempts to unite product & process randomness are deeply problematic, so CT can at best apply to process randomness
  - Random processes produce random outcomes with high probability, but are not guaranteed to do so
  - Random objects can exist independently of any process

Eagle (and others) have argued at length that the CT is problematic even for process randomness

- Deterministic randomness a key counterexample
- Randomness seems to have at least some subjective component that chance does not

Chance seems at best a sufficient condition for process randomness

Quantum Randomness as Unpredictability

Quantum randomness is often justified by an appeal to such unpredictability

"*An immediate consequence of this is objective randomness. [...] we can in principle only make a probabilistic prediction of, say, the position of a particle in a future measurement.*"[9]

To make the connection more robust, we need to flesh out the definition of (process) randomness as unpredictability

- Most crucially, need to formalise (un)predictability
- Need to reconcile subjective aspects of randomness with supposed "absolute" nature of quantum randomness
  - Even if unpredictability has a subjective component, this does not mean it isn't intrinsic, or that it can't have an objective origin
  - Can consider different *degrees* of randomness

[9] J. Kofler & A. Zeilinger. Eur. Rev. 18 (2010), p. 470.

Formalising Unpredictability

How can we formalise what it means for an event to be predictable?

- Laplace's "*intelligence qui ... connaîtrait toutes les forces dont la nature est animée et la situation respective des êtres qui la composent, si d'ailleurs elle était suffisamment vaste pour soumettre ces données à l'analyse*" is hardly physical
- Popper attempted to effectivise such an agent, requiring it to act via physical means and with finite precision.[10]
- Eagle considered a subjective model in which an agent – operating with a given theory and epistemic means – makes predictions that they use as their posterior credence
    - An event is predictable iff the agent's posterior credence can be centred on a single outcome

[10] K. Popper, Br. J. Phil. Sci. 1 (1950), 118,124.

An Effective Model of Unpredictability

We propose a model that:

- Is explicitly *effective*: the predictor must make a prediction using *computable, finite means*
  - We adopt Turing computability, as the Church-Turing thesis privileges this, but the model of computation can be varied to study degrees of effective predictability
- The predictor uses *finite information extracted locally* from the physical system *via measurement* to compute a prediction
- The degree of subjectivity can be varied by limiting (or not) the extraction capabilities of the predictor

The effectivity concerns the power of the predictor and doesn't, *a priori*, constrain the computability of the events themselves

An Effective Model of Unpredictability

More precisely, for an experiment/prediction task E, consider[11]

- An extraction technique ξ used to obtain *finite, locally accessible* information $\xi(\lambda)$ from the ontological state λ
- A computable function P_E which computes a prediction using the information extracted: i.e. $P_E(\xi(\lambda))$
- E is ξ-predictable if there exists a P_E such that, for any instantiation of E, the prediction $P_E(\xi(\lambda_i))$ is always correct
- E is predictable if there exists a ξ such that it is ξ-predictable

Note that even if a prediction is subjective (i.e., made by a particular predictor), unpredictability is objective in the sense that E is unpredictable for *any* predictor

[11] A. Abbott, C. Calude & K. Svozil, Information 6 (2015), 773.

Randomness as Unpredictability

Weak Randomness

An experiment E is Ξ-random or *(Ξ-)weakly random* if it is ξ-unpredictable for any $\xi \in \Xi$.

Strong Randomness

An experiment E is *strongly random* iff it is ξ-unpredictable for any ξ.

- Like Eagle, we permit very biased events to be random. Could generalise model to quantify randomness and require *uniformity* for maximal randomness, but not clear how reasonable this is

Chaotic Randomness

Unpredictability of chaotic systems first recognised by Poincaré:

"*Il peut arriver que de petites différences dans les conditions initiales en engendrent de très grandes dans les phénomènes finaux; une petite erreur sur les premières produirait une erreur énorme sur les derniers. La prédiction devient impossible.*"[12]

- Given a class Ξ of extractors with bounded precision, such systems can produce weakly random events
  - Not strongly random assuming classical mechanics, since a sufficiently good extractor exists for any E
  - If such a limit represents a fundamental physical restriction, would be strongly random
  - For such systems, external perturbations pose a further, eventually more important, limit

Thus, in principle, one may have deterministic, intrinsic randomness

[12] H. Poincaré, Science et Methode, Flammarion (1908), 68–69.

Quantum Randomness

Does this notion of randomness correctly explain quantum randomness?

- It is not difficult to show that quantum value indefiniteness (i.e., indeterminism) implies unpredictability and strong randomness
  - λ doesn't determine, and thus can't be used to predict, the measurement outcome

Seems to match the kind of informal account of quantum randomness mentioned earlier

- Captures difference between quantum and some forms of deterministic randomness
  - Degree of relativisation: random irrespective of predictors' means
  - Not about intrinsic vs. epistemic nature: may have intrinsic, deterministic strong randomness

Quantum Randomness

Understanding quantum randomness as process randomness via unpredictability is crucial to better judging claims about its practical applications

- Cryptography demands unpredictability against any adversary, but ideally product randomness as well

A crucial aspect of quantum randomness is not just that it is random, but that this randomness can be "device-independently verified". As pointed out by Cavalcanti and Wiseman, "*if it is impossible to signal faster than light, then it is impossible to predict the outcomes of experiments that violate Bell inequalities, even if those outcomes might be determined by an underlying hidden-variable model.*"[13]

- From measurement statistics alone, under some reasonable assumptions, one can *certify* that a quantum device is random without trusting its internal workings

[13] E. Cavalcanti & H. Wiseman, Found. Phys. 42 (2012), 1330.

Indeterminism, Chance and Randomness

Where does this leave the relationship between indeterminism, chance, and randomness?

$$\text{Chance} \implies \text{Indeterminism} \implies \text{Randomness}$$

- Indeterminism necessary, but not sufficient for chance
- Indeterminism seems to entail strong randomness

Weaker forms of randomness can clearly be consistent with determinism, but strong randomness as unpredictability does not conflate the concepts

- Deterministic (strong) randomness can potentially arise in several ways
  - Fundamental limits on the extraction power of predictors
  - Experimental outcomes that depend on an infinite amount of initial information, or on *information not locally available to a predictor*

Summary

- Common accounts of quantum randomness have been based on quantum indeterminism and invoked the Commonplace Thesis to deem measurements random
- Quantum indeterminism is well grounded in the theorems of Bell and Kochen and Specker
- The process notion of randomness applicable to individual quantum measurements can be formalised rigorously via lack of effective (i.e. computable) predictability
- This corresponds well to common claims about quantum randomness, and the advantages it provides in quantum information
- However, one must be more cautious about claims of 'true' or 'absolute' randomness

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The Kochen-Specker Theorem

Kochen and Specker showed the contradiction with determinism occurs even at the level of the Hilbert space representation, *prior* to measurement

They asked whether a hidden variable can consistently determine all measurement outcomes prior to measurement

- Noncontextuality: A measurement outcome must be independent of the other *compatible* observables measured alongside it
  - i.e. if A is compatible with both B and C, but B and C are incompatible, we should obtain the same outcome for the proposition A, whether we measure $\{A, B\}$ or $\{A, C\}$
  - A generalisation of locality

Their result applies to *individual* quantum particles (but, for technical reasons, not two-dimensional systems)

The Kochen-Specker Theorem (2)

Eigenstate-Eigenvalue Link

A system in a state $|\psi\rangle$ has a definite value of an observable property A if and only if $|\psi\rangle$ is an eigenstate of A.

The 'if' direction of the E-E Link is solid; do these results prove the 'only if'?

- Not quite: they only show that not all observables can be value definite for a state
- But possible[14] to strengthen the KS Theorem to show that every observable A must be value definite for $|\psi\rangle$ unless $|\psi\rangle$ is an eigenstate of A.

[14] A. Abbott, C. Calude & K. Svozil. J. Math. Phys. **56**, 102201 (2015).

Chaotic Randomness

Is chance a necessary condition for this type of randomness? How would this effect the status of quantum randomness?

"*Il peut arriver que de petites différences dans les conditions initiales en engendrent de très grandes dans les phénomènes finaux; une petite erreur sur les premières produirait une erreur énorme sur les derniers. La prédiction devient impossible.*"[15]

"*Il faut donc bien que le hasard soit autre chose que le nom que nous donnons à notre ignorance.*"[16]

- Even if this 'unpredictability' is not due to chance, the role of the 'interval of measure' is essential and thus the unpredictability is nonetheless intrinsic

[15] H. Poincaré, Science et Methode, Flammarion (1908), 68–69.

[16] H. Poincaré, Calcul des probabilités, Gauthier-Villars (1912).

Chaotic Randomness

For a given prediction task for a chaotic system (e.g., after a fixed time):

- Assuming classical mechanics, a sufficiently good extractor exists and thus renders the task predictable, and hence not random
- But given a fixed limit on measurement accuracy (a restriction on the extractors), such tasks may exhibit *relativised* unpredictability
- If such there was a fundamental, physical such limit, such processes *would be random*
    - For such systems, external perturbations pose a further, eventually more important, limit

Thus, in principle, one may have deterministic, intrinsic randomness.

What is Measurement?

Even if the Born rule is to be interpreted as representing real chance, a lingering problem remains: QM described unitary, reversible, dynamics, while measurement thus involves irreversible, acausal change. How does this change occur?

"*On the information-theoretic interpretation, the "big" measurement problem is a pseudo-problem. If the universe is genuinely indeterministic and measurement outcomes are intrinsically random, then it isn't possible to provide a dynamical explanation of how a system produces a definite outcome when it's measured—that's what it means for the measurement outcomes to be intrinsically random.*"[17]

But if QM is truly complete, it should be applicable to the measurement apparatus itself, which appears to still pose a problem

[17] J. Bub, Entropy 17 (2015), p. 7377.

The Measurement Problem

- Gisin[18] argues that to avoid the conclusion that "there are several kinds of stuffs out there, i.e. physical dualism, some stuff that respects the superposition principle and some that doesn't, or there are special configurations of atoms and photons where the superposition principle breaks down", QM must be extended to describe the dynamical collapse of the quantum state
- Everettians argue that measurement is subjective, a result of the observer becoming entangled with the system being measured
- Similarly, Quantum Bayesianism (QBism) takes the view that the Born rule should be interpreted in a strictly Bayesian sense
    - Quantum randomness must be understood differently in such frameworks where one does not have indeterminism; can it still be given a consistent meaning?

[18] N. Gisin, arXiv:1701.08300 (2017).

Kochen-Specker Definitions

Definition

A *value assignment function* on $\mathcal{O}$ is a partial two-valued function $v : \mathcal{O} \times \mathcal{C}_{\mathcal{O}} \to \{0, 1\}$.

Definition

An observable $P \in \mathcal{O}$ is noncontextual under v if, for all $C, C' \in \mathcal{C}_{\mathcal{O}}$ with $P \in C, C'$, we have $v(P, C) = v(P, C')$.

Definition

A value assignment function v is admissible if for every context $C \in \mathcal{C}_{\mathcal{O}}$:

(a) if there exists a $P \in C$ with $v(P) = 1$, then $v(P') = 0$ for all $P' \in C \setminus \{P\}$;

(b) if there exists a $P \in C$ with $v(P') = 0$ for all $P' \in C \setminus \{P\}$, then $v(P) = 1$.

Kochen-Specker and Bell Theorems

Kochen-Specker Theorem

Let $n \geq 3$ and $|\psi\rangle, |\phi\rangle \in \mathbb{C}^n$ be states such that $0 < |\langle\psi|\phi\rangle| < 1$. Then there is a finite set of observables $\mathcal{O}$ containing P_ψ and P_ϕ for which there is no admissible value assignment function on $\mathcal{O}$ such that $v(P_\psi) = 1$ and P_ϕ is value definite.

Bell-CHSH Inequality

For two parties A and B, who can each choose between two measurements labelled $x, y = \pm 1$ producing outputs $a, b = \pm 1$, any locally realist model must satisfy

$$|C(a,b|0,0) + C(a,b|0,1) + C(a,b|1,0) - C(a,b|1,1)| \leq 2,$$

where $C(a,b|x,y) = \sum_{a,b}(-1)^{ab} p(a,b|x,y)$ is the correlation between a and b on inputs x and y.