Self-testing Quantum Supermaps
with an application to the Quantum Switch

Victor Barizien[1], Alastair A. Abbott[2], Cyril Branciard[3], Pavel Sekatski[4] and Jean-Daniel Bancal[1]

[1]Université Paris Saclay, CEA, CNRS, Institut de physique théorique, 91191 Gif-sur-Yvette, France
[2]Univ. Grenoble Alpes, Inria, 38000 Grenoble, France
[3]Univ. Grenoble Alpes, CNRS, Grenoble INP, Institut Néel, 38000 Grenoble, France
[4]Département de Physique Appliquée, Université de Genève, 1211 Genève, Switzerland

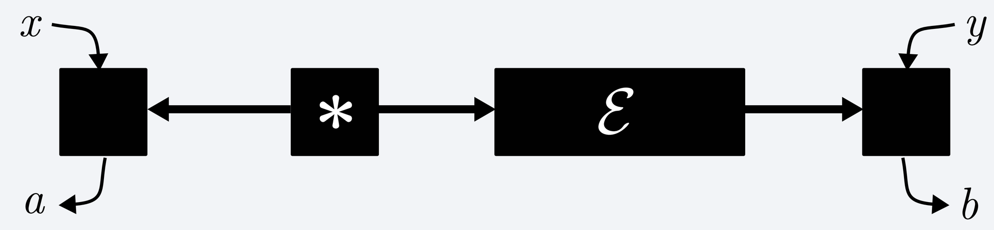
Motivation

- Ongoing debate regarding experiments implementing the Quantum Switch and the ability to certify its causal indefiniteness in a device-independent setting
- Use self-testing techniques to certify a Quantum Switch in a black-box way
- Framework of self-testing needs to be generalised to quantum supermaps

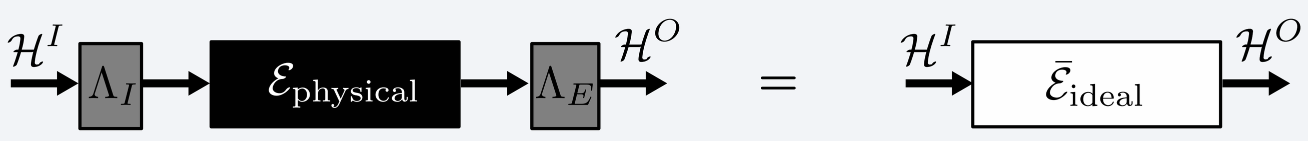
Self-testing

- Determine the most accurate physical description of a device possible without prior knowledge of the internal workings of the apparatuses involved
- Black-box setting: devices used are uncharacterised but their connections are trusted
- Characterise device from the observed correlations and the network structure

Example: self-testing a quantum channel [1]



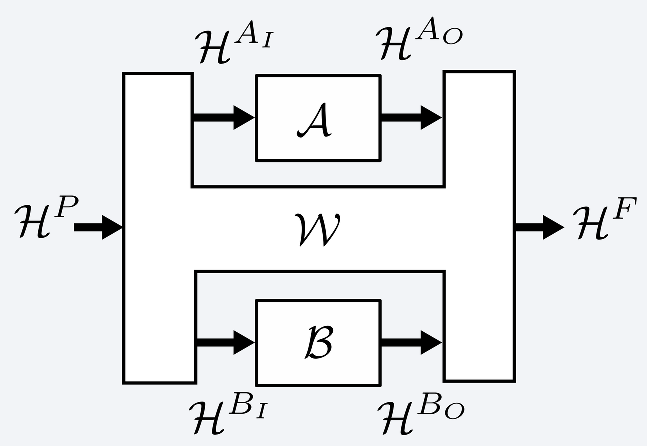
- Can only self-test up to DOFs undetectable in the black-box setting
  - Local change of basis, ancillas, etc. undetectable through correlations alone
- Desired correlations $p(a,b|x,y) \implies \exists \Lambda_I, \Lambda_O$ such that:



Quantum supermaps

- Higher-order quantum operation: transforms quantum maps into quantum maps

$$(\mathcal{A},\mathcal{B}) \mapsto \mathcal{W}(\mathcal{A},\mathcal{B})$$



- Represented in Choi picture as a process matrix:

$$W \in \mathcal{L}(\mathcal{H}^P \otimes \mathcal{H}^{A_I} \otimes \mathcal{H}^{A_O} \otimes \mathcal{H}^{B_I} \otimes \mathcal{H}^{B_O} \otimes \mathcal{H}^F)$$

- Choi matrices

$$A \in \mathcal{L}(\mathcal{H}^{A_I} \otimes \mathcal{H}^{A_O}) \text{ and } B \in \mathcal{L}(\mathcal{H}^{B_I} \otimes \mathcal{H}^{B_O})$$

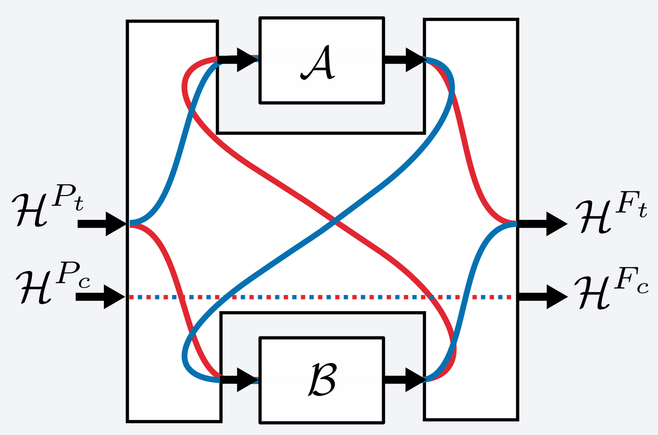
of $\mathcal{A}$ and $\mathcal{B}$ combined into

$$W * (A \otimes B) \quad \in \mathcal{L}(\mathcal{H}^P \otimes \mathcal{H}^F)$$

The Quantum Switch

- Some quantum supermaps describe causally indefinite compositions of $\mathcal{A}$ and $\mathcal{B}$
  - Causal nonseparability: $\mathcal{W} \neq q\mathcal{W}^{A \prec B} + (1-q)\mathcal{W}^{B \prec A}$
- This property can be tested in both device-dependent (DD) and, sometimes, device-independent (DI) settings

Example: The Quantum Switch



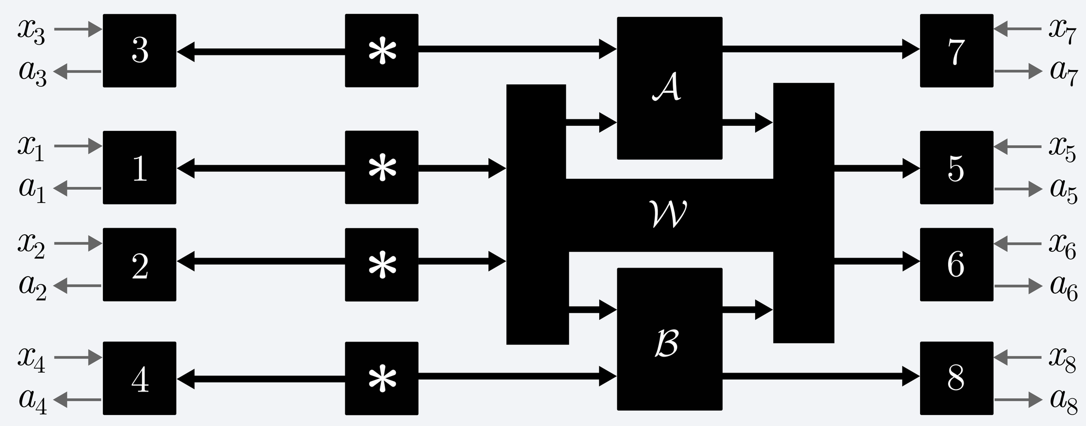
- Coherent quantum control of $\mathcal{B} \circ \mathcal{A}$ and $\mathcal{A} \circ \mathcal{B}$
- $W_{\text{QS}} = |w_{\text{QS}}\rangle\langle w_{\text{QS}}|$ with

$$|w_{\text{QS}}\rangle = |0\rangle^{P_c} |\mathbb{1}\rangle\rangle^{P_t A_I} |\mathbb{1}\rangle\rangle^{A_O B_I} |\mathbb{1}\rangle\rangle^{B_O F_t} |0\rangle^{F_c} + |1\rangle^{P_c} |\mathbb{1}\rangle\rangle^{P_t B_I} |\mathbb{1}\rangle\rangle^{B_O A_I} |\mathbb{1}\rangle\rangle^{A_O F_t} |1\rangle^{F_c},$$

where $|\mathbb{1}\rangle\rangle = \sum_i |i,i\rangle \propto |\phi^+\rangle$
- The causal nonseparability of the Quantum Switch can be witnessed in a network DI setting [2, 3]
- But existing DI tests don't self-test the Quantum Switch

Self-testing scheme

- Both the supermap and the devices plugged into it must be treated as black boxes



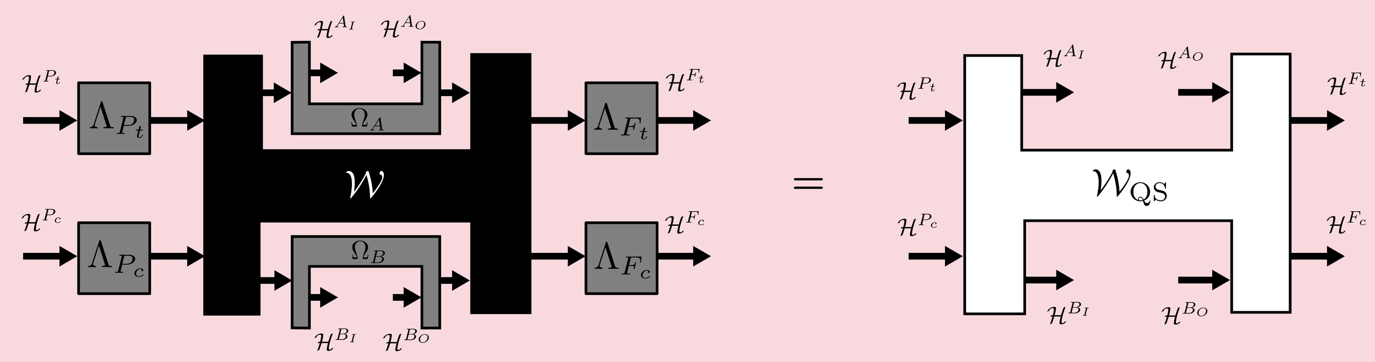
Reference scenario:
- Sources are maximally entangled qubit states $|\phi^+\rangle$
- $\mathcal{A}$ and $\mathcal{B}$ are SWAP gates
- The Quantum Switch then acts as a controlled swap on (half of) four $|\phi^+\rangle$ states:

$$|\overline{\psi}_{\text{out}}\rangle = \frac{1}{4}\sum_{i,j,k=0}^{1} \left(|00\rangle^{26} |ijk\rangle^{134} |kij\rangle^{578} + |11\rangle^{26} |ijk\rangle^{134} |jki\rangle^{578} \right)$$

- Idea: self-test $|\overline{\psi}_{\text{out}}\rangle$ with multiple permuted CHSH tests
  - Parties 1,2,3,4 measure $M_0 = \sigma_z$, $M_1 = \sigma_x$
  - Parties 5,7,8 measure $M_0 = \frac{\sigma_z+\sigma_x}{\sqrt{2}}$, $M_1 = \frac{\sigma_z-\sigma_x}{\sqrt{2}}$
  - Party 6 measures $M_0 = \sigma_z$, $M_1 = \sigma_x$, $M_2 = \frac{\sigma_z+\sigma_x}{\sqrt{2}}$

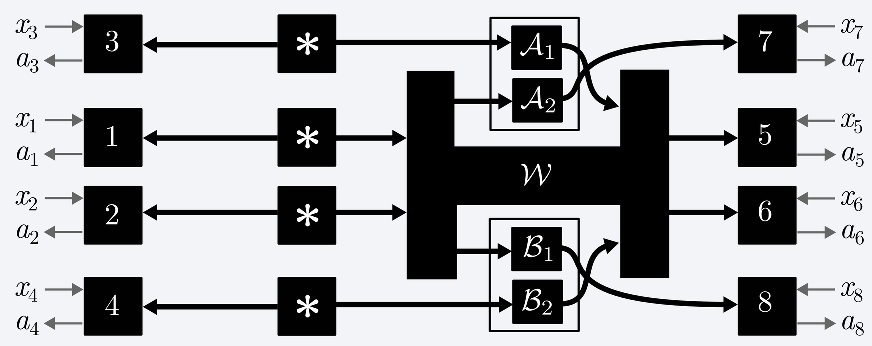
Theorem: Self-testing the input and output states
- Given these statistics, $\exists \Lambda_i : \mathcal{L}(\mathcal{H}_i) \to \mathbb{C}^2$ such that $\otimes_{i=1}^{8} \Lambda_i(\rho_{\text{out}}) = |\overline{\psi}_{\text{out}}\rangle\langle\overline{\psi}_{\text{out}}|$
- The same measurements self test ρ_{in}: $\exists \Lambda_i'$ such that $\otimes_{i=1}^{8} \Lambda_i'(\rho_{\text{in}}) = |\phi^+\rangle\langle\phi^+|^{\otimes 4}$

Theorem: Self-testing the Quantum Switch
- Given these statistics, $\exists \Lambda_{P_c}, \Lambda_{P_t}, \Lambda_{F_c}, \Lambda_{F_t}, \Omega_A, \Omega_B$ such that:

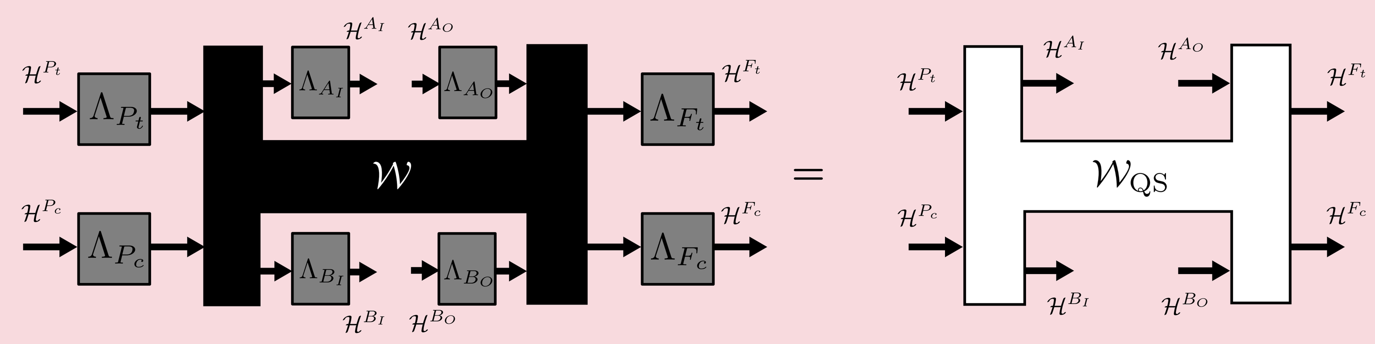


A finer self-testing statement

- Self-test is up to local embedding combs at slots for $\mathcal{A}$ and $\mathcal{B}$
  - Impossible to have a finer statement if $\mathcal{A}$ and $\mathcal{B}$ considered whole black boxes
- Observation: In the black-box setting, we control network connectivity
  - Connect separate devices to $\mathcal{H}^{A_I}, \mathcal{H}^{A_O}, \mathcal{H}^{B_I}, \mathcal{H}^{B_O}$
  - $\mathcal{A}$ and $\mathcal{B}$ often considered closed laboratories: Alice, Bob plug devices as desired



Theorem: Self-testing the Quantum Switch up to local channels
- Given the same statistics, $\exists \Lambda_{P_c}, \Lambda_{P_t}, \Lambda_{A_I}, \Lambda_{A_O}, \Lambda_{B_I}, \Lambda_{B_O}, \Lambda_{F_c}, \Lambda_{F_t}$ such that:



Conclusions and open questions

- Can identify degrees of freedom on which the implemented supermap is precisely $\mathcal{W}_{\text{QS}}$
- Self-testing approach applicable to general supermaps, e.g. quantum combs with 1 slot
- Which causally nonseparable quantum supermaps can be self-tested?
- Simplify setting and relax assumptions

References and acknowledgements

[1] P. Sekatski, J.-D. Bancal, S. Wagner, and N. Sangouard, Phys. Rev. Lett. **121**, 180505 (2018).
[2] T. van der Lugt, J. Barrett, and G. Chiribella, Nat. Commun. **14**, 5811 (20223).
[3] H. Dourdent, A. A. Abbott, I. Šupić, and C. Branciard, (2023), arXiv:2308.12760 [quant-ph].

This work is supported by the Agence Nationale de la Recherche projects EPiQ (ANR-22-PETQ-0007, part of Plan France 2030), and TaQC (ANR-22-CE47-0012).