Device-Independent Quantification of Quantum Resources

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Motivation and Goals

- Quantum resources (states, measurements, channels, ...) provide advantages that can be operationally quantified
- Quantifying a given resource typically requires well characterised states and/or measurements to probe the resource
- What resources can be characterised in a device independent way?
- **Goal:** use techniques from self-testing to certify and quantify any resourceful object in a black-box setting

Quantum Resources

We consider resources with convex free sets:

- **States:** entanglement, steerability, non-Gaussianity, magic, ...
- **Measurements:** incompatibility, non-projective-simulability, ...
- **Channels:** non-entanglement-breaking and non-incompability-breaking channels, thermal operations, ...

We focus on channel resources

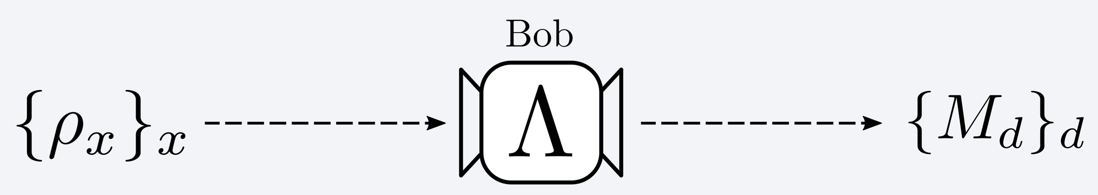
- Resourcefulness of Λ w.r.t. a free set F quantified with the generalised robustness:

$$R_F(\Lambda) = \min_{\tilde{\Lambda}} \left\{ t \geq 0 \;\middle|\; \frac{\Lambda + t\tilde{\Lambda}}{1+t} \in F \right\}$$

Resource Quantification with Input-Output Games [2]

- $R_F(\Lambda)$ related to operational advantage in an input-output game $\mathcal{G} = (\mathcal{E}, \mathcal{M}, \Omega)$:

$$\mathcal{E} = \{p(x)\rho_x\}_x \quad \text{(input state ensemble)}$$
$$\mathcal{M} = \{M_d\}_d \quad \text{(a POVM)}$$
$$\Omega = \{w_{x,d}\}_{x,d} \quad \text{(score)}$$



$$P(\Lambda, \mathcal{G}) = \sum_{x,d} p(x)\, \omega_{x,d}\, \mathrm{tr}[\Lambda(\rho_x) M_d] \qquad \text{(payoff)}$$

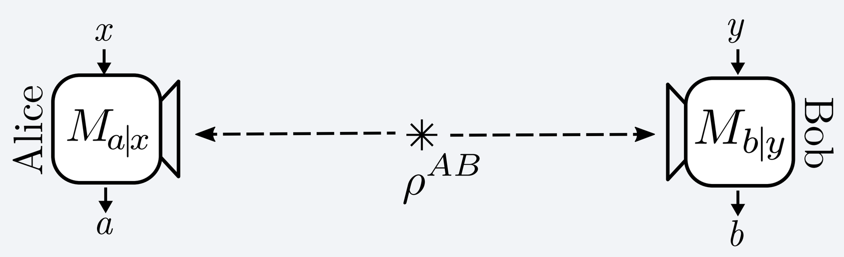
- For well-normalised input-output games, payoff a resource can give is directly related to its robustness:

$$1 + \mathcal{R}_F(\Lambda) = \max_{\mathcal{G}} P(\Lambda, \mathcal{G})$$

- Device dependent: Must trust $\mathcal{E}$ and $\mathcal{M}$!

Self-testing [3]

Certify exact form of a state and measurements from correlations $p(a, b|x, y)$

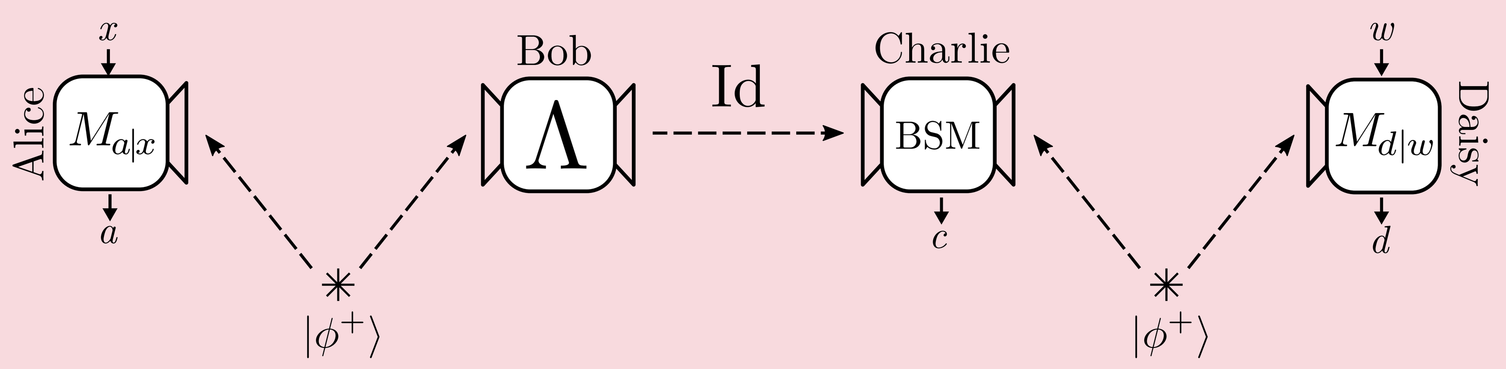


- E.g.: maximal violation of a Bell inequality can certify:
  - $\rho \simeq |\phi^+\rangle\langle\phi^+|$, Alice and Bob measure Pauli X, Y, Z
- Certification up to local isometries and complex conjugate

Reference Scenario & Protocol

Use self-testing to characterise, device-independently:

- remote preparation of pure states states $\{\rho_x\}_x$
- arbitrary measurement $\{M_d\}_d$



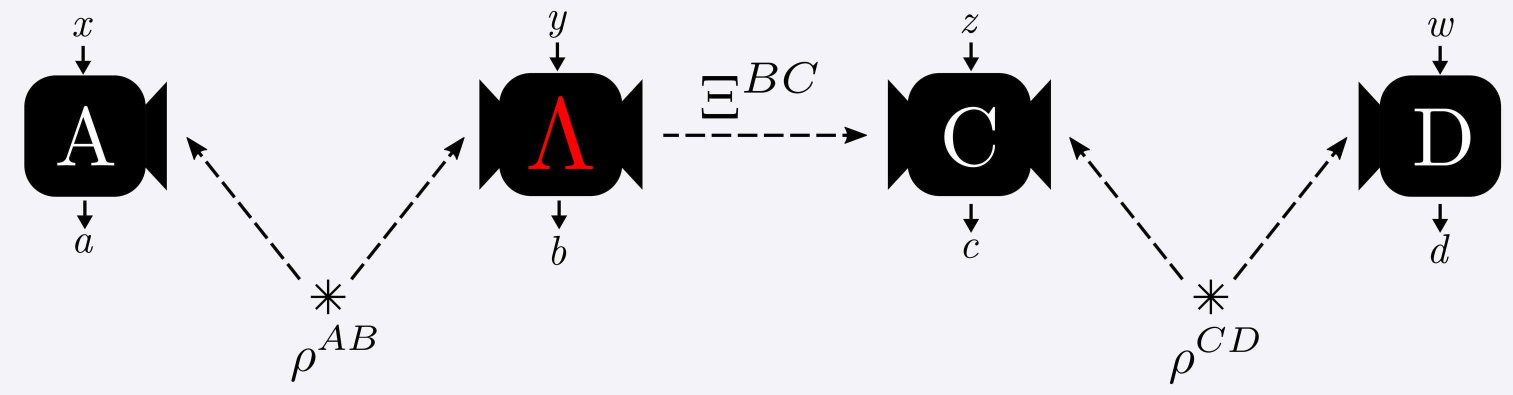
1. On input x, Alice remotely prepares ρ_x for Bob by performing a suitable measurement on her share of $|\phi^+\rangle^{AB}$.
2. Bob applies Λ to ρ_x and sends $\Lambda(\rho_x)$ to Charlie via the identity channel Id.
3. Charlie performs a Bell-state measurement (BSM) on $\Lambda(\rho_x)$ and his share of $|\phi^+\rangle^{CD}$, teleporting to Daisy the state $U_c\Lambda(\rho_x)U_c^\dagger$.
4. Daisy measures $\{M_{d|w}\}_d = \{U_w M_d U_w^\dagger\}_d$ on the teleported state; for $w = c$ this is equivalent to measuring $\{M_d\}_d$ on $\Lambda(\rho_x)$.

$$P(\Lambda, \mathcal{G}) = \sum_{x,d} p(x)\, \omega_{x,d} \sum_{c,w} \frac{1}{p(0|x)} p(0, c, d|x, w)\, \delta_{c,w}$$

Device-independent Quantification Protocol

Add extra inputs to self-test (up to local isometries):

- Maximally entangled states ρ^{AB} and ρ^{CD}
- Pauli X, Y, Z measurements for Alice and Daisy
- Identity channel Ξ^{BC}
- Bell-state measurement for Charlie



- Causal structure: Alice and Daisy don't know when Bob applies Λ
- Self-tests that we are playing the game $\mathcal{G}$ on an effective subspace
  - (up to a correlated partial transpose on Alice and Daisy)

Quantification Statement

- The statistics on "quantification rounds" give the payoff of an effective channel

$$\Lambda^{\text{eff}} = h_0 \Lambda_0^{\text{eff}} + h_1 \tilde{\Lambda}_1^{\text{eff}}$$

as

$$P(\Lambda^{\text{eff}}, \mathcal{G}) = \sum_{x,d} p(x)\, \omega_{x,d} \sum_{c,w} \frac{1}{p(0|x)} p(0, c, d|x, \Diamond, w)\, \delta_{c,w}$$

  - Indistinguishability of correlated Alice-Daisy conjugation:
    - $\tilde{\Lambda}_1^{\text{eff}}(\rho) = \Lambda_1^{\text{eff}}(\rho^*)^*$ (conjugate channel)
    - $h_0, h_1 \in \{0, 1\}$, $h_0 + h_1 = 1$
  - Λ_i^{eff} can be directly related to the "total physical channel" from Bob to Daisy:

$$\mathcal{T}^{C \to D} \circ \Xi^{BC} \circ \Lambda$$

    - Take into account "junk states" and local isometries used to "extract" the effective channel
    - Λ_0^{eff} and Λ_1^{eff} differ in junk states arising from self-testing isometries
- DI certification that experiment contains an effective channel with payoff $P(\Lambda^{\text{ext}}, \mathcal{G})$ on the game $\mathcal{G}$

Relation to Physical Channel

For "well-behaved" resources:

- **Resource certification:** If Λ is resourceless, so is Λ^{eff}
- **Quantification bound:** $P(\Lambda^{\text{eff}}, \mathcal{G}) \leq \max_{\mathcal{G}'} P(\Lambda, \mathcal{G}')$
  - i.e., $R_F(\Lambda^{\text{eff}}) \leq R_F(\Lambda)$
  - Lower bound on resourcefulness of physical channel Λ

Resource must satisfy certain preconditions:

- Can't be increased by local channels
- Insensitive to channel conjugation

Examples

- Non-entanglement-breaking and non-incompatibility-breaking channels are faithfully quantified in this way
- Can be significantly simplified for state or measurement resources
  - E.g., input-output games $\longrightarrow$ state-discrimination games
  - Complements known DI certification of all entangled states [4]
- We likewise obtain a fully black-box certification of any sets of incompatible measurements

Conclusions and Open Questions

- DI certification of any well-behaved resourceful channel
- Correlations quantify the resourcefulness of implemented channel
- Causal network structure of protocol important to its success
- Full characterisation of which resources can be quantified in this way?
- Explicit procedure to extract use of the effective channel Λ^{eff}?

References

[1] A. A. Abbott, N. Brunner, I. Šupić, and R. Uola, in preparation (2022).
[2] R. Uola, T. Kraft, and A. A. Abbott, Phys. Rev. A **101**, 052306 (2019).
[3] I. Šupić and J. Bowles, Quantum **4**, 337 (2020).
[4] J. Bowles, I. Šupić, D. Cavalcanti, and A. Acín, Phys. Rev. Lett. **121**, 180503 (2018).