Communication Through Coherent Control of Quantum Channels

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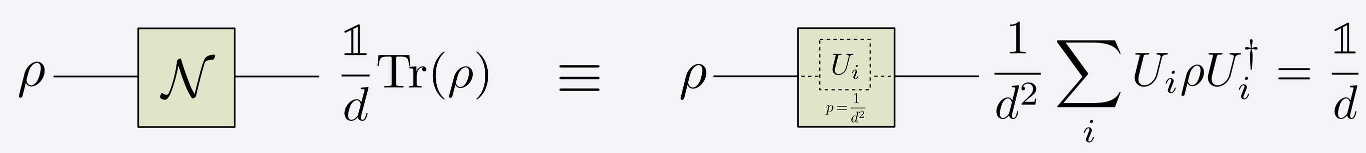
Motivation

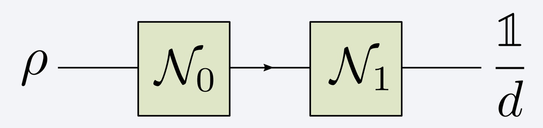
- In [2] it was shown that communication through two noisy channels can be enhanced if the order in which they are used is coherently controlled in a "quantum switch"
- We seek to understand whether indefinite causal order is the origin of this "causal activation"
- We show that simply controlling coherently between two noisy channels leads to a similar activation phenomenon and study such coherent coherent control of channels more generally

Communication Through Noisy Channels

- Consider two parties communicating through some noisy network
- In extreme case, model as completely depolarising channel
  - Can view as random application of orthogonal unitaries with $p = \frac{1}{d^2}$

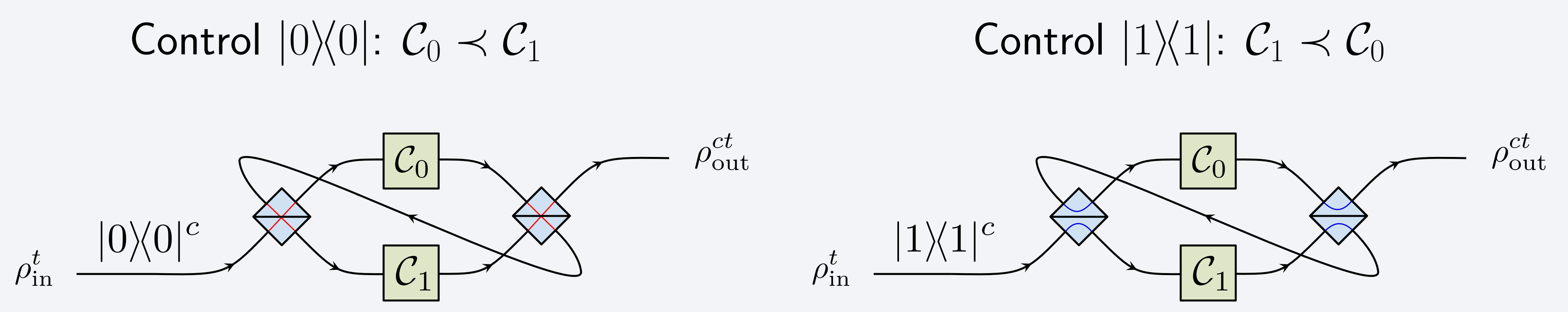


- In the case that their communication must be routed through two noisy regions:

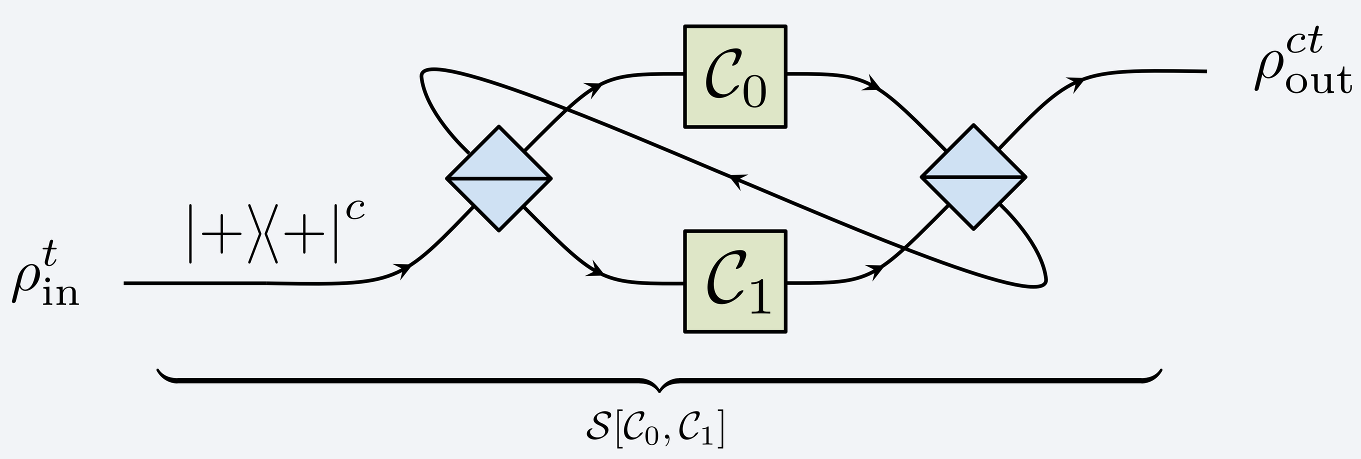


Causal Activation with the Quantum Switch [2]

- The quantum switch is a new way to compose channels in a superposition of different orders



- Control $|+\rangle\langle +|$: coherent superposition of the two causal orders
  - Induces a new global channel $\mathcal{S}[\mathcal{C}_0, \mathcal{C}_1]$



- The output of the global channel is

$$\mathcal{S}[\mathcal{N}_0, \mathcal{N}_1](\rho_{\text{in}}^t) = \frac{\mathbb{1}^c}{2} \otimes \frac{\mathbb{1}^t}{d} + \frac{1}{2}\big[\,|0\rangle\langle 0|\,1^c + |1\rangle\langle 1|\,0^c\big] \otimes \frac{1}{d^2}\rho_{\text{in}}^t$$

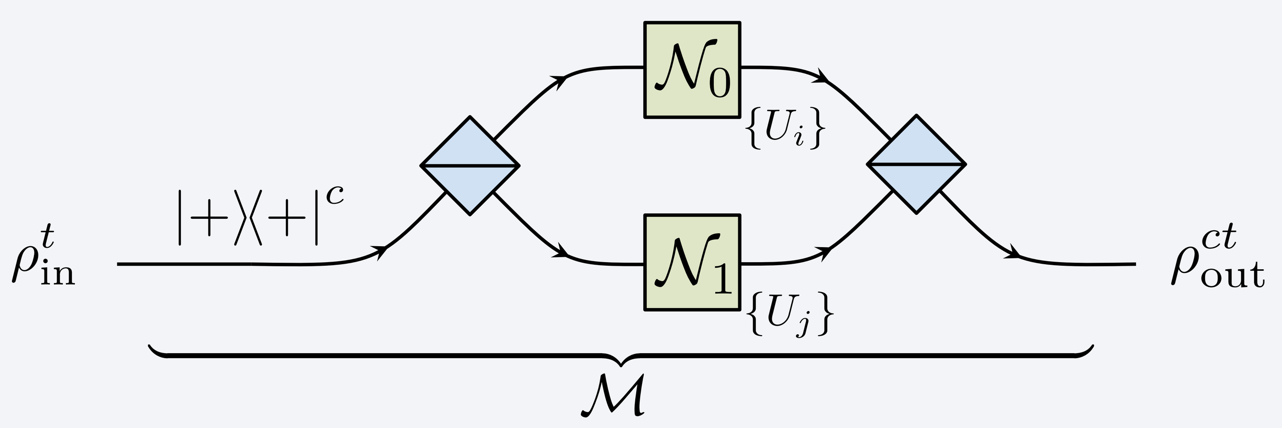
When two completely depolarising channels are placed in the quantum switch, information is transmitted to the output ρ_{out}^{ct} [2]

- For qubits, Holevo information is $\chi(\mathcal{S}[\mathcal{N}_0, \mathcal{N}_1]) = -\frac{3}{8} - \frac{5}{8}\log_2\frac{5}{8} \approx 0.05$
- Quantum communication capacity also activated for communication through dephasing channels [3]

Should we attribute this to the indefinite causal order of the quantum switch?

Half a Quantum Switch but Twice as Good [1]

- Consider the simplified scenario of coherently controlling between $\mathcal{N}_0$ and $\mathcal{N}_1$



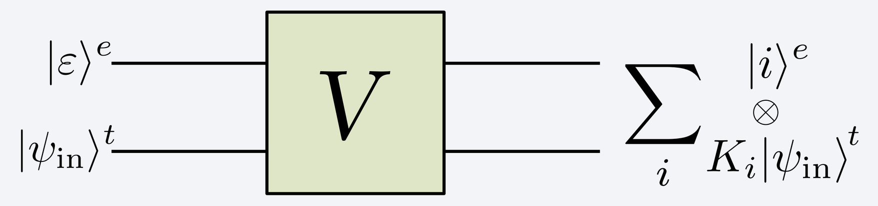
- To calculate ρ_{out}^{ct}, consider randomisation over choice of $\{U_i\}_i$ for each channel

$$\rho_{\text{out}}^{ct} = \frac{\mathbb{1}^c}{2} \otimes \frac{\mathbb{1}^t}{d} + \frac{1}{2}\big[|0\rangle\langle 1|^c + |1\rangle\langle 0|^c\big] \otimes T\rho_{\text{in}}^t T^\dagger, \quad \text{with } T := \frac{1}{d^2}\sum_i U_i$$

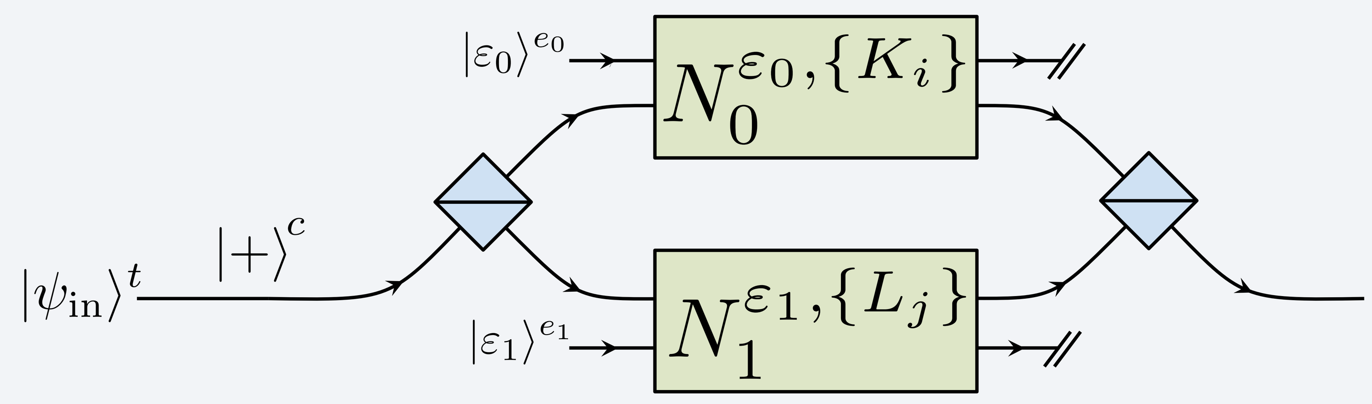
- $T\rho_{\text{in}}^t T^\dagger \neq 0$ and depends on ρ_{in}^t: capacity activated with no causal indefiniteness
- For qubits (where U_i are Pauli unitaries) Holevo information is $\chi(\mathcal{M}) \approx 0.12$
- Quantum capacity can again be activated through dephasing channels
- But note that T matrix depends on the choice of U_i!

Implementation Dependence in Coherent Control of Channels

- More generally, for any Kraus representation $\{K_i\}_i$ of $\mathcal{N}$ can represent channel as unitary Stinespring dilation before tracing out environment:



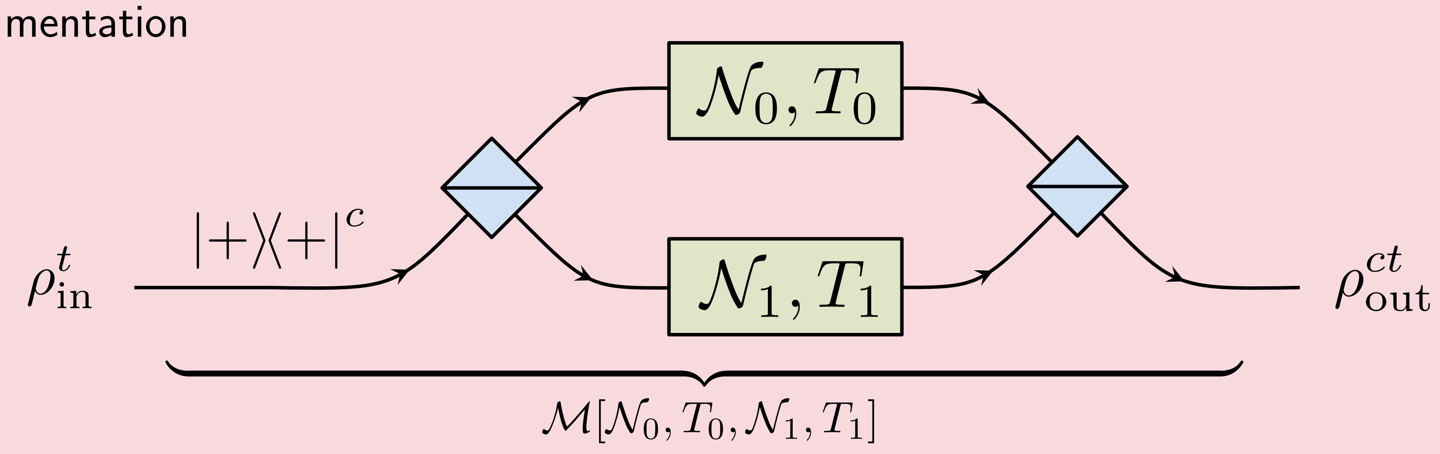
- Most general output ρ_{out}^{ct} can be thus calculated as



$$\rho_{\text{out}}^{ct} = \frac{\mathbb{1}^c}{2} \otimes \frac{\mathbb{1}^t}{d} + \frac{1}{2}\big[|0\rangle\langle 1|^c \otimes T_0\rho_{\text{in}}^t T_1^\dagger + |1\rangle\langle 0|^c \otimes T_1\rho_{\text{in}}^t T_0^\dagger\big]$$

with $T_0 := \sum_i \langle\varepsilon_0|i\rangle K_i$ and $T_1 := \sum_j \langle\varepsilon_1|j\rangle L_j$

Description of coherently controlled channels must be supplemented by **transformation matrices** describing the relevant information about their implementation



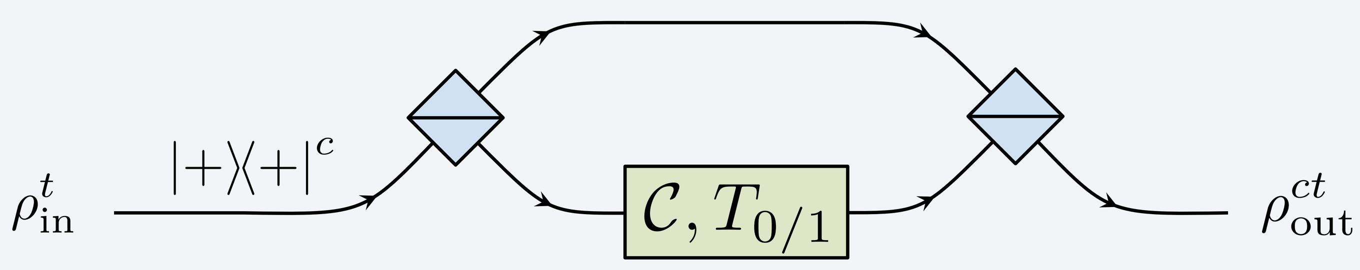
- Can characterise precisely the transformation matrices obtainable for any channel
- For depolarising channel, $\mathrm{Tr}[TT^\dagger] \le \frac{1}{d}$
  - Maximal capacity is $\chi(\mathcal{M}[\mathcal{N}_0, T_0, \mathcal{N}_1, T_1]) = \frac{1}{4}\log_2\frac{5}{4}$ (≈ 0.12 for qubits)

Distinguishing Implementations of a Channel

- Can use implementation dependence to discriminate between two implementations of a channel $\mathcal{C}$ with transformation matrices T_0 and T_1
- Optimal probability of discrimination is

$$p_{\text{discrim.}} = \frac{1}{2}\big(1 + \tfrac{1}{2}\|T_0 - T_1\|_2\big)$$

  - For two implementations of depolarising channels, this is $\frac{1}{2}(1 + \frac{1}{\sqrt{d}}) \approx 0.85$



Conclusions and Open Questions

- Coherent control of channels can be used to help improve communication through noisy channels
- Induced global channel depends on *implementation* of the controlled channels
- How to disentangle role of coherent control and indefinite causal order? [4]
- Adds to call to generalise standard quantum circuit paradigm to include wider class of experimentally conceivable situations including coherent control

References

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